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Oscillation Theorems for Third Order Nonlinear Delay Differential Equation with "Maxima"

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Abstract. In this paper we obtain oscillation criteria for the third order delay differential equation with "maxima" of the form

$$\left(a(t)\left(\left(b(t)(x'(t))^{\alpha}\right)'\right)^{\beta}\right)' + q(t)\max_{[\sigma(t),t]} x^{\gamma}(s) = 0,\ t \geq t_0$$

via comparison with the oscillatory behavior of first order differential equations. Some examples are given to illustrate the main results.

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Keywords and Phrases: Oscillation; third order; differential equation with "maxima".

1.Introduction

This paper deals with the oscillation of third order nonlinear delay differential equation with maxima of the form

(1.1.) $$\left(a(t)\left(\left(b(t)(x'(t))^{\alpha}\right)'\right)^{\beta}\right)' + q(t)\max_{[\sigma(t),t]} x^{\gamma}(s) = 0,\ t \geq t_0,$$

subject to the following conditio

(H1) $a(t), b(t)$ and $q(t) \in C([t_0,\infty),(0,\infty))$;

(H2) α, β and γ are quotient of odd positive integer;

(H3) $\sigma(t) \in C^1([t_0,\infty),R)$, $\sigma(t) \leq t$ for $t \geq t_0$ and $\lim_{t\to\infty}\sigma(t) = \infty$;

By a solution of equation (1.1), we mean a function $x(t)$ defined for all $t \geq t_x \geq t_0$ such that $x(t), b(t)(x'(t))^{\alpha}$, $a(t)\left(\left(b(t)(x'(t))^{\alpha}\right)'\right)^{\beta}$ are continuous and differentiable for all $t \geq t_x$ and satisfies equation (1.1) for all $t \geq t_x$ and satisfy $\sup\{|x(t)| : t \geq T\} > 0$ for any $T \geq t_x$. It will be assumed that equation (1.1) has nontrivial solutions exist for all $t_0 \geq 0$. A solution of equation (1.1) is called oscillatory if it has infinitely many zeros, otherwise it is called nonoscillatory.

In the last few years, the oscillation and asymptotic behavior of differential equations with "maxima" received considerable attention because of the fact that they appear in the study of systems with automatic regulation, and automatic control of various technical systems. It often occurs that the law of regulation depends on maximum values of some regulated state parameter over certain intervals, see [4, 9].

In [1, 2, 3, 5], the authors study the oscillatory behavior of solutions of equation (1.1) when $\alpha = 1$ or

$\beta = 1$, and therefore in this paper we consider equation (1.1) which include many results considered in [1, 2, 3, 5] as special cases.

The purpose of this paper is to investigate the oscillatory behavior of solutions of equation (1.1) with the cases

(1.2) $$\int_{t_0}^{\infty} \frac{1}{b^{1/\alpha}(t)} dt = \infty, \ \int_{t_0}^{\infty} \frac{1}{a^{1/\beta}(t)} dt = \infty$$

and

(1.3) $$\int_{t_0}^{\infty} \frac{1}{b^{1/\alpha}(t)} dt < \infty, \ \int_{t_0}^{\infty} \frac{1}{a^{1/\beta}(t)} dt < \infty.$$

The results obtained in this paper improvement and extend that of in [1, 2, 3, 5], and many known results.

3. Oscillation Results

In this section, we state and prove our main results. Without loss of generality, we consider only positive solutions of equation (1.1) since the proof for the negative solution is similar. We begin with the following lemmas which will paly an important role in proving the main results. Define

$$B(t,t_0) = \int_{t_0}^{t} \frac{1}{b^{1/\alpha}(s)} ds,$$

$$A_1(t) = \int_{t}^{\infty} \frac{1}{b^{1/\alpha}(s)} ds,$$

$$A_2(t) = \int_{t}^{\infty} \frac{1}{a^{1/\alpha}(s)} ds.$$

Lemma 2.1 *Let there is a* $T_1 \ge t_0$ *such that* $\sigma(t) > T_1$ *for* $t \ge T > T_1$ *and*

(C_1) *either*

(2.1) $$\int_{t_0}^{\infty} \frac{1}{a^{1/\beta}(t)} dt = \infty,$$

or

(2.2) $$\int_{T}^{\infty} \frac{1}{a(t)} \left(\int_{T}^{t} q(s) A_2^{\gamma}(s) B^{\gamma}(s,T) ds \right)^{1/\beta} dt = \infty$$

(C_2) *either*

(2.3) $$\int_{t_0}^{\infty} \frac{1}{b^{1/\alpha}(t)} dt = \infty$$

or

(2.4) $$\int_{T}^{\infty} \frac{1}{b^{1/\alpha}(t)} \left(\int_{T}^{t} \frac{1}{a^{1/\beta}(s)} \left(\int_{T}^{s} q(u) A_1^{\gamma}(\sigma(u)) du \right)^{1/\beta} ds \right)^{1/\alpha} dt = \infty$$

hold. If x *be an eventually positive solution of equation (1.1), then* x *satisfies one of the following two cases:*

(I) $x'(t) > 0, \left(b(t)(x'(t))^{\alpha} \right) > 0$ *for all* $t \ge T$ *;*

(II) $x'(t) < 0, \left(b(t)(x'(t))^{\alpha} \right) > 0$ *for all* $t \ge T$ *.*

Proof. Let $x(\sigma(t)) > 0$ for all $t \ge t_1 \ge t_0$. From equation (1.1), we have

$$\left(a(t)\left(\left(b(t)(x'(t))^{\alpha}\right)'\right)^{\beta}\right)' = -q(t)\max_{[\sigma(t),t]} x^{\gamma}(s) < 0 \ \ for\ all\ t \ge t_1.$$

Then $a(t)\left(b(t)(x'(t))^{\alpha}\right)'$ strictly decreasing for all $t \ge t_1$ and thus $x'(t)$ and $b(t)(x'(t))^{\alpha}$ are eventually of one sign. We show that $b(t)(x'(t))^{\alpha} > 0$ for all $t \ge t_1$. Now assume $b(t)(x'(t))^{\alpha} \le 0$ for all $t \ge t_1$ and we have two cases:

Case1. Let there exists $t_2 \ge t_1$ sufficiently large, such that $x'(t) > 0$ and $\left(b(t)(x'(t))^{\alpha}\right)' < 0$ for $t \ge t_2$.

Case2. Let there exists $t_2 \ge t_1$ sufficiently large, such that $x'(t) < 0$ and $\left(b(t)(x'(t))^{\alpha}\right)' < 0$ for $t \ge t_2$.

Case(1). In the case we have $b(t)(x'(t))^{\alpha}$ is strictly decreasing for $t \ge t_2$ and there is a constant $M < 0$ such that

$$a(t)\left(\left(b(t)(x'(t))^{\alpha}\right)'\right)^{\beta} < M,\ t \ge t_2.$$

Dividing by $a(t)$ and then integrating from t_2 to t, we obtain

$$b(t)(x'(t))^{\alpha} \le b(t_2)(x'(t_2))^{\alpha} + M^{1/\beta}\int_{t_2}^{t}\frac{1}{a^{1/\beta}(s)}ds.$$

Letting $t \to \infty$ and using (2.1), we have $x'(t) \to -\infty$, which is a contradiction.

Next consider (2.2). Then, we have

$$x(t) \ge x(t) - x(t_3) = \int_{t_3}^{t} b^{-1/\alpha}(s)\left(b(s)(x'(s))^{\alpha}\right)^{1/\alpha} ds$$

$$\ge \left(b(t)(x'(t))^{\alpha}\right)^{1/\alpha}\int_{t_3}^{t}\frac{1}{b^{1/\alpha}(s)}ds,\ t \ge t_3.$$

From equation (1.1) and the last inequality, we have

(2.5) $$0 = \left(a(t)(y'(t))^{\beta}\right)' + q(t)\max_{[\sigma(t),t]} x^{\gamma}(s) \ge \left(a(t)(y'(t))^{\beta}\right)' + q(t)y^{\gamma}(t)B^{\gamma}(t,t_3),$$

where $y(t) = b(t)(x'(t))^{\alpha}$. It is clear that $y(t) > 0$ and $y'(t) < 0$, and it follows that

$$-y'(t) \ge -\frac{a^{1/\beta}(t_3)y'(t_3)}{a^{1/\beta}(t)},\ t \ge t_3.$$

Integrating the last inequality from t to ∞ from t to ∞, we obtain

(2.6) $$y(t) \ge K_1 A_2(t),\ t \ge t_4 \ge t_3,$$

where $K_1 = -a^{1/\beta}(t_3)y'(t_3) > 0$. Integrating (2.5) from t_4 to t and using (2.6), we obtain

$$\int_{t_4}^{t} q(s)K_1^{\gamma}A_2^{\gamma}(s)B^{\gamma}(s,t_3)ds \le a(t_4)\left(y'(t_4)\right)^{\beta} - a(t)\left(y'(t)\right)^{\beta},$$

or

$$\left(\frac{K_1^{\gamma}}{a(t)}\int_{t_4}^{t} q(s)A_2^{\gamma}(s)B^{\gamma}(s,t_3)ds\right)^{1/\beta} \le -y'(t).$$

Again integrating from t_4 to ∞, we get

$$K_1^{\gamma/\beta}\int_{t_4}^{\infty}\left(\frac{1}{a(t)}\int_{t_4}^{t}q(s)A_2^{\gamma}(s)B^{\gamma}(s,t_3)ds\right)^{1/\beta}dt \le y(t_4) < \infty$$

which contradicts (2.2).

Case(2). In this case, we have

$$b(t)(x'(t))^{\alpha} \le b(t_2)(x'(t_2))^{\alpha} = K < 0.$$

Dividing the above inequality by $b(t)$ and integrating from t_2 to t, we obtain

$$x(t) \le x(t_2) + K^{1/\alpha}\int_{t_2}^{t}\frac{1}{b^{1/\alpha}(s)}ds.$$

Letting $t \to \infty$, then condition (2.3) implies that $x(t) \to -\infty$, which is a contradiction. Next, assume condition (2.4) is satisfied. One can choose $t_3 \ge t_2$ with $\sigma(t) \ge t_2$ for all $t \ge t_3$ such that

$$x(\sigma(t)) \ge -\left(b_1(\sigma(t))(x'(\sigma(t)))^{\alpha}\right)^{1/\alpha}A_1(\sigma(t))$$
$$\ge K_2A_1(\sigma(t)),\ t \ge t_3,$$

where $K_2 = -\left(b_1(\sigma(t))(x'(\sigma(t)))^{\alpha}\right)^{1/\alpha} > 0$. Then from equation (1.1), we have

$$\left(a(t)\left(\left(b(t)(x'(t))^{\alpha}\right)\right)^{\beta}\right) = -q(t)\max_{[\sigma(t),t]}x^{\gamma}(s)$$
$$= -q(t)x^{\gamma}(\sigma(t))$$
$$\le -Lq(t)A_1^{\gamma}(\sigma(t)),$$

where $L = K_2^{\gamma}$. Integrating the last inequality from t_3 to t, we obtain

$$\left(b(t)(x'(t))^{\alpha}\right) \le L^{1/\beta}\frac{1}{a^{1/\beta}(t)}\left(\int_{t_3}^{t}q(s)A_1^{\gamma}(\sigma(s))ds\right)^{1/\beta}.$$

Again integrating the above integrating from t_3 to t, we get

$$x'(t) \le Kb^{-1/\alpha}(t)\left(\int_{t_3}^{t}\frac{1}{a^{1/\beta}(s)}\left(\int_{t_3}^{s}q(u)A^{\gamma}(\sigma(u))du\right)^{1/\beta}ds\right)^{1/\alpha},$$

where $K = L^{1/\alpha\beta}$. Once again integrating the last inequality from t_3 to t, we obtain

$$K\int_{t_3}^{t}\left(\frac{1}{b^{1/\alpha}(s)}\left(\int_{t_3}^{s}\frac{1}{a^{1/\beta}(u)}\left(\int_{t_3}^{u}q(v)A^{\gamma}(\sigma(v))dv\right)^{1/\beta}du\right)^{1/\alpha}\right)ds \le x(t_3) < \infty$$

Letting $t \to \infty$ in the above inequality, we obtain a contradiction with (2.4). Thus, we have $\left(b(t)(x'(t))^{\alpha}\right) > 0$ for $t \ge t_1$ and hence $x'(t) > 0$ or $x'(t) < 0$ for $t \ge t_1$. Then proof is now complete.

Lemma 2.2 *Let conditions* (C_1) *and* (C_2) *be hold. Let* $x(t)$ *be an eventually positive solution of equation (1.1) for all* $t \ge t_0$ *and suppose that Case(II) of Lemma 2.1 holds. If*

(2.7) $$\int_{t_0}^{\infty}\frac{1}{b^{1/\alpha}(s)}\left(\int_t^{\infty}\frac{1}{a^{1/\beta}(s)}\left(\int_s^{\infty}q(u)du\right)^{1/\beta}ds\right)^{1/\alpha}dt=\infty$$

then $x(t)\to 0$ *as* $t\to\infty$.

Proof. Let $x(t)$ be a positive solution of equation (1.1) and there is a $t_1\ge t_0$ such that $x(\sigma(t))>0$ for $t\ge t_1$. Since $x(t)$ is decreasing, we get $\lim_{t\to\infty}x(t)=\ell\ge 0$. Assume $\ell>0$, then $x(\sigma(t))\ge\ell$ for all $t\ge t_2\ge t_1$. Integrating equation (1.1) from t to ∞, we find

$$a(t)\left(\left(b(t)(x'(t))^{\alpha}\right)\right)^{\beta}\ge\int_t^{\infty}q(s)\max_{[\sigma(s),s]}x^{\gamma}(u)ds\ge\int_t^{\infty}q(s)x^{\gamma}(\sigma(s))ds.$$

Since $x(t)$ is decreasing. Dividing the last inequality by $a(t)$ and then integrating from t to ∞, we get

$$-x'(t)\ge\frac{\ell^{1/\alpha\beta}}{b^{1/\alpha}(t)}\left[\int_t^{\infty}\left(\frac{1}{a(s)}\int_s^{\infty}q(u)du\right)^{1/\beta}ds\right]^{1/\alpha}.$$

Again integrating from t_2 to ∞, we see that

$$x(t_2)\ge\ell^{1/\alpha\beta}\int_{t_2}^{\infty}\frac{1}{b^{1/\alpha}(t)}\left[\int_t^{\infty}\left(\frac{1}{a(s)}\int_s^{\infty}q(u)du\right)^{1/\beta}ds\right]^{1/\alpha}dt.$$

which is a contradiction with (2.7). Thus $\lim_{t\to\infty}x(t)=0$. The proof is now complete.

Theorem 2.1 *Let conditions* $(C_1),(C_2)$ *and* $\sigma'(t)>0$ *be hold for all* $t\ge t_0$, *and there exists a differential function* $\tau(t)$ *such that*

(2.8) $$\tau'(t)\ge 0,\tau(t)>t,\ \ and\ \ \sigma(\tau(\tau(t)))<t.$$

If both the first order delay equations

(2.9) $$y'(t)+q(t)\left(\int_{t_0}^{\sigma(t)}\frac{1}{b^{1/\alpha}(s)}\left(\int_{t_0}^{s}\frac{1}{a^{1/\beta}(u)}du\right)^{1/\alpha}ds\right)^{\gamma}y^{\gamma/\alpha\beta}(\sigma(t))=0,$$

and

(2.10) $$z'(t)+\frac{1}{b^{1/\alpha}(t)}\left(\int_t^{\tau(t)}\frac{1}{a^{1/\beta}(s)}\left(\int_s^{\tau(s)}q(u)du\right)^{1/\beta}ds\right)^{1/\alpha}z^{\gamma/\alpha\beta}(\eta(t))=0,$$

when $\eta(t)=\sigma(\tau(\tau(t)))$, *are oscillatory, then every solution of equation (1.1) is oscillatory.*

Proof. Let $x(t)$ be a nonoscillatory solution of equation (1.1). Then, without loss of generality, there is a $t_1\ge t_0$ such that $x(t)>0, x(\sigma(t))>0$ for all $t\ge t_1$. Choose $t_2\ge t_1$ sufficiently large so that two cases of Lemma 2.1 hold.

Case(I). In this case, we have

$$b(t)(x'(t))^{\alpha}=b(t_2)(x'(t_2))^{\alpha}+\int_{t_2}^{t}a^{-1/\beta}(s)y^{1/\beta}(s)ds\ge y^{1/\beta}(t)\int_{t_2}^{t}a^{-1/\beta}(s)ds,$$

where $y(t)=a(t)\left(\left(b(t)(x'(t))^{\alpha}\right)\right)^{\beta}>0.$ It follows that

$$x'(t) \geq \frac{y^{1/\alpha\beta}(t)}{b^{1/\alpha}(t)}\left(\frac{1}{a^{1/\alpha}(s)}ds\right)^{1/\alpha}.$$

Integrating the last inequality from t_2 to t, we get

$$x(t) \geq \int_{t_2}^{t}\frac{y^{1/\alpha\beta}(s)}{b^{1/\alpha}(s)}\left(\int_{t_2}^{s}\frac{1}{a^{1/\beta}(u)}du\right)^{1/\alpha}ds \geq y^{1/\alpha\beta}(t)\int_{t_2}^{t}\frac{1}{b^{1/\alpha}(s)}\left(\int_{t_2}^{s}\frac{1}{a^{1/\beta}(u)}du\right)^{1/\alpha}ds.$$

Let there exists $t_3 \geq t_2$ such that $\sigma(t) \geq t_2$ for all $t \geq t_3$, then

$$x(\sigma(t)) \geq y^{1/\alpha\beta}(\sigma(t))\int_{t_2}^{\sigma(t)}\frac{1}{b^{1/\alpha}(s)}\left(\int_{t_2}^{s}\frac{1}{a^{1/\beta}(u)}du\right)^{1/\alpha}ds,\ t \geq t_3.$$

From equation (1.1), we have

$$-y'(t) = q(t)\max_{[\sigma(t),t]}x^{\gamma}(s) = q(t)x^{\gamma}(t) \geq q(t)x^{\gamma}(\sigma(t))$$

(2.11) $$\geq q(t)y^{\gamma/\alpha\beta}(\sigma(t))\left(\int_{t_2}^{\sigma(t)}\frac{1}{b^{1/\alpha}(s)}\left(\int_{t_2}^{s}\frac{1}{a^{1/\beta}(u)}du\right)^{1/\alpha}ds\right).$$

Integrating the above inequality from t to ∞, we get

$$y(t) \geq \int_{t}^{\infty}q(s)y^{\gamma/\alpha\beta}(\sigma(s))\left(\int_{t_2}^{\sigma(s)}\frac{1}{b^{1/\alpha}(v)}\left(\int_{t_2}^{v}\frac{1}{a^{1/\beta}(u)}du\right)^{1/\alpha}dv\right)^{\gamma}ds.$$

The function $y(t)$ is clearly strictly decreasing and hence by Theorem 1 of [8] there exists a positive solution of equation (2.9) which contradicts that the equation (2.9) is oscillatory.

Case(II). Integrating equation (1.1) from t to $\tau(t)$, we obtain

$$a(t)\left(\left(b(t)(x'(t))^{\alpha}\right)\right)^{\beta} \geq \int_{t}^{\tau(t)}q(s)\max_{[\sigma(s),s]}x^{\gamma}(u)ds \geq \int_{t}^{\tau(t)}q(s)x^{\gamma}(\sigma(s))ds.$$

From (2.8), we obtain

$$\left(b(t)(x'(t))^{\alpha}\right) \geq \frac{1}{a^{1/\beta}(t)}x^{\gamma/\beta}(\sigma(\tau(t)))\left(\int_{t}^{\tau(t)}q(s)ds\right)^{1/\beta}.$$

Integrating again the last inequality from t to $\tau(t)$, we have

$$-b(t)(x'(t))^{\alpha} \geq \int_{t}^{\tau(t)}\frac{1}{a^{1/\beta}(s)}x^{\gamma/\beta}(\sigma(\tau(s)))\left(\int_{s}^{\tau(s)}q(u)du\right)^{1/\beta}ds.$$

or

(2.12) $$-x'(t) \geq \frac{x^{\gamma/\alpha\beta}(\eta(t))}{b^{1/\alpha}(t)}\left(\int_{t}^{\tau(t)}\frac{1}{a^{1/\beta}(s)}\left(\int_{s}^{\tau(s)}q(u)du\right)^{1/\beta}ds\right)^{1/\alpha}.$$

Integrating the above inequality from t to ∞, we obtain

$$x(t) \geq x^{\gamma/\alpha\beta}(\eta(t))\int_{t}^{\infty}\frac{1}{b^{1/\alpha}(t)}\left(\int_{t}^{\tau(t)}\frac{1}{a^{1/\beta}(s)}\left(\int_{s}^{\tau(s)}q(u)du\right)^{1/\beta}ds\right)^{1/\alpha}dt.$$

In view of Theorem 1 in [8] there exists a positive solution of equation (2.10) which contradicts that equation (2.10) is oscillatory. This completes the proof.

By combining Case(I) in the proof of Theorems 2.1 with Lemma 2.2, we obtain the following theorem.

Theorem 2.2 *Let conditions (2.7), (C_1) and (C_2) be hold. If the first order delay equation (2.9) is oscillatory, then every solution $x(t)$ of equation (1.1) is either oscillatory or tends to zero as $t \to \infty$.*

Remark 2.1 *Let $b(t) = 1$ and $\alpha = 1$, then Theorem 2.1 and Theorem 2.2 are reduced to that of in [1, 2].*

Corollary 2.1 *Let $\frac{\gamma}{\alpha\beta} = 1$, and the hypotheses of Theorem 2.1 hold. If*

$$\liminf_{t\to\infty} \int_{\sigma(t)}^{t} q(s)\left(\int_{t_0}^{\sigma(s)} \frac{1}{b^{1/\alpha}(u)}\left(\int_{t_0}^{u} \frac{1}{a^{1/\beta}(v)}dv\right)^{1/\alpha}\right)^{\gamma} ds > \frac{1}{e}, \tag{2.13}$$

and

$$\liminf_{t\to\infty} \int_{\eta(t)}^{t} \frac{1}{b^{1/\alpha}(s)}\left(\int_{s}^{\tau(s)} \frac{1}{a^{1/\beta}(u)}\left(\int_{u}^{\tau(u)} q(v)dv\right)^{1/\alpha}\right)^{\gamma} ds > \frac{1}{e} \tag{2.14}$$

respectively, then every solution of equation (1.1) is oscillatory.

Proof. Proceeding as in the proof of Theorem 2.1, we have (2.11) and (2.12) with $\frac{\gamma}{\alpha\beta} = 1$. By condition (2.13) and (2.14) and Theorem 2.1.1 of [6], the inequalities (2.11) and (2.12) have no positive solution which a contradiction. This completes the proof.

Corollary 2.2 *Let $0 < \frac{\gamma}{\alpha\beta} < 1$, and the hypotheses of Theorem 2.1 hold. If*

$$\int_{t_0}^{\infty} q(t)\left(\int_{t_0}^{\sigma(t)} \frac{1}{b^{1/\alpha}(s)}\left(\int_{t_0}^{s} \frac{1}{a^{1/\beta}(u)}du\right)^{1/\alpha} ds\right)^{\gamma} dt = \infty, \tag{2.15}$$

and

$$\int_{t_0}^{t} \frac{1}{b^{1/\alpha}(t)}\left(\int_{t}^{\tau(t)} \frac{1}{a^{1/\beta}(s)}\left(\int_{s}^{\tau(s)} q(u)du\right)^{1/\beta}\right)^{1/\alpha} dt = \infty, \tag{2.16}$$

then every solution of equation (1.1) is oscillatory.

Proof. Proceeding as in the proof of Theorem 2.1, we have inequalities (2.11) and (2.12) with $0 < \frac{\gamma}{\alpha\beta} < 1$. By condition (2.15) and (2.16) and Theorem 3.9.3 of [6], the inequalities (2.11) and (2.12) have no positive solution. This contradiction completes the proof.

3 Examples

In this section, we present two examples to illustrate the main results.

Example 3.1 Consider the third order differential equation

$$\left(\left(t\left(\frac{1}{t^2}(x'(t))^{1/3}\right)\right)^3\right)' + t \max_{[t^{1/5},t]} x(s) = 0, \; t \ge 1. \tag{3.1}$$

Here $a(t)=t, b(t)=\frac{1}{t^2}, \alpha=\frac{1}{3}, \beta=3, \gamma=1, q(t)=t$, and $\sigma(t)=t^{1/5}$. A simple calculation shows that $\sigma(t)=t^{1/5}<t, \sigma'(t)>0$ and

$$\int_1^\infty \frac{1}{a^{1/\beta}(t)}dt=\int_1^\infty \frac{1}{t^{1/3}}dt=\infty,\ \int_1^\infty \frac{1}{b^{1/\alpha}(t)}dt=\int_1^\infty t^6 dt=\infty.$$

It is easy to see that all condition (2.7) holds, and equation (2.9) reduces to

$$y'(t)+t\left(c_1t^{9/5}+c_2t^{5/3}+c_3t^{23/15}-c_4t^{7/5}\right)y\left(t^{1/5}\right)=0, \tag{3.2}$$

where c_1, c_2, c_3, and c_4 are constants. By Theorem 2.1.1 of [6] guarantees oscillation of equation (3.2) provided that

$$\lim_{t\to\infty} inf \int_{t^{1/5}}^t s\left(c_1s^{9/5}+c_2s^{5/3}+c_3s^{23/15}-c_4s^{7/5}\right)ds=\infty>\frac{1}{e}$$

and according to Theorem 2.2 every solution of equation (3.1) is either oscillatory or tends to zero as $t\to\infty$.

Example 3.2 Consider the third order differential equation

$$\left(t^3\left(t^6x'(t)\right)\right)+t^{12}\max_{[t/2,t]} x(s)=0,\ t\geq 1. \tag{3.3}$$

Here $a(t)=t^3, b(t)=t^6, \alpha=\beta=\gamma=1, \sigma(t)=\frac{t}{2}, q(t)=t^{12}$. A simple calculation shows that $\sigma(t)=\frac{t}{2}<t, \sigma'(t)>0$, $\lim_{t\to\infty}\sigma(t)=\lim_{t\to\infty}\frac{t}{2}=\infty$, and $\int_1^\infty \frac{1}{t^6}dt<\infty, \int_1^\infty \frac{1}{t^3}dt<\infty$. It is easy to see that conditions (2.2), (2.4) and (2.7) hold. Further, equation (2.9) reduces to

$$z'(t)+\frac{t^{12}(t^7-112t^2+320)}{35t^7}z(t/2)=0. \tag{3.4}$$

By Theorem 2.1.1 of [6] guarantees of oscillation of (3.4) provided that

$$\lim_{t\to\infty} inf \int_{t/2}^t \frac{s^5(s^7-112s^2+320)}{35}ds>\frac{1}{e}$$

and according to Theorem 2.2, every solution of equation (3.4) is either oscillatory or tends to zero as $t\to\infty$.

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