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Doubt Fuzzy BOI-ideal of BOI-algebra

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Abstract:

The aim of this paper is to introduce the notion of doubt fuzzy BOI-ideal of BOI-algebra and to study their properties. The homomorphic image and the inverse image of a doubt fuzzy BOI-ideal is studied well. The Cartesian product of doubt fuzzy BOI-ideal is introduced and discussed.

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1. Introduction:

Imai and Iséki presented two classes of abstract algebras: BCK-algebra and BCI algebra ([11], [12]). These algebras have been extensively studied since their introduction. In ([9], [10]) Hu and Li introduced a wide class of abstract algebras: BCH-algebra which is a generalization of the concept of BCK and BCI-algebras and studied some of the properties of this algebra. In 2002, Neggers and Kim [16] introduced a new notion, called B-algebras, and obtained several results. Jun, Roh, and Kim [14] introduced the notion of BH-algebra which was a generalization of B/BCI/BCK-algebras and generalized some theories of BCI-algebras. In 2007, Walendziak [18] introduced a new notion, called a BF-algebra, which is a generalization of B-algebra. Moreover, C. B. Kim and H. S. Kim [15] introduced BG-algebra as a generalization of B-algebra. In [17] Neggers, Ahn and Kim introduced Q-algebra, and also in [1] Ahn and Kim introduced QS-algebras, which are a generalization of BCH/BCI/BCK-algebras. In [2], Bandaru introduced a new notion called BRK-algebra which is a generalization of BCK/BCI/BCH/Q/QS/BM-algebras. Fuzzy sets, which were introduced by Zadeh [19], deal with possibilistic uncertainty, connected with the imprecision of states, perceptions, and preferences. In ([4], [5]), we introduced the fuzzification of BRK-ideal of BRK-algebra and investigated some relevant characteristics, see also [6]. In [7] we introduce a new notion, called a BOI-algebra which is a generalization of BCI/BCH –algebras and is related to several classes of algebras such as Q/BG/BH/BF-algebras. We consider the fuzzification of BOI-ideal of BOI-algebra. Several fundamental properties that are related to fuzzy BOI-ideals are investigated, see also [8]. Jun [13] defined a doubt fuzzy subalgebra, doubt fuzzy ideal, doubt fuzzy implicative ideal, and doubt fuzzy prime ideal in BCI-algebras, and got some results about it. The purpose of this paper is to apply the concept of doubt fuzzy set to BOI-ideal of BOI-algebra. The notion of doubt fuzzy BOI-ideal is defined, and a lot of properties are investigated. We discussed the homomorphic image and the inverse image of doubt fuzzy BOI-ideal. The Cartesian product of doubt fuzzy BOI-ideal of BOI-algebra is studied. Several theories and basic properties that are related to doubt fuzzy BOI-ideal are investigated. In section 5, we conclude and present some topics for future research.

2. Preliminaries:

Definition 2.1 [7]. A BOI-algebra $(X;*,0)$ (i.e., a nonempty set X with a binary operation "$*$" and a constant 0) satisfying the following axioms:

(M_1) $x * x = 0$,

(M_2) $x * (x * y) = y$,

(M_3) $(x * y) * z = (x * z) * y$, for all $x, y, z \in X$.

In X, we can define a binary relation "$\leq$" by $x \leq y$ if and only if $x * y = 0$.

Proposition 2.2 [7]. If $(X; *, 0)$ is a BOI-algebra, then following conditions hold:

(1) $x * 0 = x$,

(2) $0 * (x * y) = y * x$,

(3) $(z * x) * (z * y) = y * x$

(4) $(x * z) * (y * z) = x * y$,

(5) $(x * y) * (0 * y) = x$,

(6) $x * (x * (x * y)) = x * y$,

(7) $(x * y) * x = 0 * y$,

(8) $x * (y * z) = (x * y) * (0 * z)$.

Example 2.3[7]. Let $X = \{0,1,2\}$. Define $*$ on X as the following table:

$*$	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

Then $(X;*,0)$ is a BOI-algebra.

Definition 2.4 [7]. A nonempty subset I of a BOI-algebra X is called a subalgebra of X if $x * y \in I$ for all $x, y \in I$

Definition 2.5 [7]. A non-empty subset M of a BOI-algebra $(X; *, 0)$is called a BOI-ideal if for any $x, y, z \in X$:

B$_1$) $0 \in M$,

B$_2$) $x * y \in M \ and \ (x * z) * y \in M \ imply \ z \in M$.

Obviously, $\{0\}$ and X are ideals of a BOI-algebra X. we call $\{0\}$ and X the zero ideal and the trivial ideal of X, respectively. An ideal M is said to be proper if $M \neq X$.

Definition 2.6[19]. Let X be a set. A fuzzy set μ in X is a function $\mu : X \to [0,1]$.

For simplicity, we will use $x \vee y$ for $\max(x, y)$, and $x \wedge y$ for $\min(x, y)$.

Definition 2.7 [7]. Let $(X;*,0)$ be a BOI-algebra. A fuzzy set μ in X is called a fuzzy BOI-ideal of X if it satisfies:

F$_1$) $\mu(0) \geq \mu(x)$,

F$_2$) $\mu(z) \geq \mu(x * y) \wedge \mu\big((x * z) * y\big), for\ all\ x, y, z \in X.$

Definition 2.8 [7]. Let $(X;*,0)$ be a BOI-algebra. A fuzzy set μ in X is called a fuzzy BOI-subalgebra of X if it satisfies:

S$_1$) $\mu(x * y) \geq \mu(x) \wedge \mu(y), for\ all\ x, y \in X.$

Example 2.9[7]. Let $X = \{0,1,2,3\}$ Define $*$ on X as the following table:

$*$	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	2
3	3	2	1	0

Then $(X; *, 0)$ is a BOI-algebra, $\{0,1\}$ is a subalgebra and BOI-ideal of X.

Definition 2.10 [Homomorphism of BOI-algebra][7]. Let $(X;*,0)$ and $(Y;*',0')$ be BOI-algebras. A mapping $f: X \to Y$ is said to be a homomorphism if $f(x*y) = f(x)*' f(y)$, for all $x, y \in X$.

Proposition 2.11[7]. Let $(X;*,0)$ and $(Y;*',0')$ be BOI-algebras, and the mapping $f: X \to Y$ be a homomorphism of BOI-algebras, then the $Kerf$ is a BOI-ideal.

3. Doubt Fuzzy BOI-ideal of BOI-algebra

Definition 3.1. Let $(X; *, 0)$ be a BOI-algebra. A fuzzy set β in X is called a doubt fuzzy BOI-ideal of X if it satisfies:

(O$_1$) $\beta(0) \leq \beta(x)$,

(O$_2$) $\beta(z) \leq \beta(x*y) \vee \beta\big((x*z)*y\big)$ for all $x, y \in X$.

Example 3.2[7]. Let $X = \{0,1,2,3\}$. Define "$*$" on X as the following table:

$*$	0	1	2	3
0	0	3	2	1
1	1	0	3	2
2	2	1	0	3
3	3	2	1	0

Then $(X;*,0)$ is a BOI-algebra. Define a fuzzy set $\beta: X \longrightarrow [0,1]$ by $\beta(0) = s_1$, $\beta(1) = \beta(3) = \beta(2) = s_2$, where $s_1, s_2 \in [0,1]$ with $s_1 < s_2$, the routine calculation gives that β is a doubt fuzzy BOI-ideal of BOI-algebra.

Proposition 3.3. Let β be a doubt fuzzy BOI-ideal of BOI-algebra$X$. If $x \leq y$, then $\beta(x) \leq \beta(0*y)$ for all $x, y \in X$.

Proof. Let β be a doubt fuzzy BOI-ideal of a BOI-algebra X. For any $x, y \in X$ so that $x \leq y$.

Since $x \le y$, then $x * y = 0$.

$$\beta(x) \le \beta(x*y) \vee \beta((x*x)*y) = \beta(0) \vee \beta(0*y) = \beta(0*y)$$

Hence $\beta(x) \le \beta(0*y)$.

Proposition 3.4. Let β be a doubt fuzzy BOI-ideal of BOI-algebra X. If $x \le y$, then $\beta(0*x) \le \beta(x)$ for all $x, y \in X$.

Proof. Let β be a doubt fuzzy BOI-ideal of a BOI-algebra *X*. For any $x, y \in X$ such that $x \le y$

Since $x \le y$ then $x * y = 0$. $\beta(0*x) \le \beta(x*y) \vee \beta\left((x*(0*x))*y\right)$

$$= \beta(0) \vee \beta((x*y)*(0*x))$$

$$= \beta(0) \vee \beta(0*(0*x)) = \beta(0) \vee \beta(x) = \beta(x)$$

Hence $\beta(0*x) \le \beta(x)$.

Theorem 3.5. A fuzzy subset β of a BOI-algebra *X* is a fuzzy BOI-ideal of *X* if and only if $\bar{\beta}$ is a doubt fuzzy BOI-ideal of *X*.

Proof. Let β be a fuzzy BOI-ideal of a BOI-algebra *X*, and let $x, y, z \in X$.

Since $\beta(0) \ge \beta(x)$.

$$1 - \beta(0) \le 1 - \beta(x)$$

then $\bar{\beta}(0) \le \bar{\beta}(x)$.

Also $\beta(z) \ge \beta(x*y) \wedge \beta((x*z)*y)$

$$1 - \beta(z) \le 1 - \langle \beta(x*y) \wedge \beta((x*z)*y) \rangle$$

$$\bar{\beta}(z) \le \langle 1 - \beta(x*y) \vee 1 - \beta((x*z)*y) \rangle$$

then $\bar{\beta}(z) \le \bar{\beta}(x*y) \vee \bar{\beta}((x*z)*y)$

So, $\bar{\beta}$ is a doubt fuzzy BOI-ideal of *X*.
Now let $\bar{\beta}$ is a doubt fuzzy BOI-ideal of BOI-algebra *X*, and let $x, y, z \in X$.

Since $\bar{\beta}(0) \le \bar{\beta}(x)$

$$1 - \bar{\beta}(0) \ge 1 - \bar{\beta}(x)$$

then $\beta(0) \ge \beta(x)$.

Also $\bar{\beta}(z) \le \bar{\beta}(x*y) \vee \bar{\beta}((x*z)*y)$

$$1 - \bar{\beta}(z) \ge 1 - \langle \bar{\beta}(x*y) \vee \bar{\beta}((x*z)*y) \rangle$$

$$\beta(z) \ge \langle 1 - \bar{\beta}(x*y) \wedge 1 - \bar{\beta}((x*z)*y) \rangle$$

then $\beta(z) \ge \beta(x*y) \wedge \beta((x*z)*y)$.

Then β is a fuzzy BOI-ideal of a BOI-algebra X.

Theorem 3.6. Let β be a doubt fuzzy BOI-ideal of BOI-algebra X. Then the set $\beta_X = \{x, y \in X | \beta(x * y) = \mu(0)\}$ is a BOI-ideal.

Proof. Clearly, $0 \in \beta_X$.

Let $x, y \in \beta_X$ be such that $(x * y) \in \beta_X$ and $((x * z) * y) \in \beta_X$.

Then $\beta(x * y) = \beta((x * z) * y) = \beta(0)$.

$\beta(z) \leq \langle \beta(x * y) \vee \beta((x * z) * y) \rangle \Longrightarrow \beta(z) \leq \beta(0) \vee \beta(0) \Longrightarrow \beta(z) \leq \beta(0)$.

Also by O_1, we have $\beta(z) \geq \beta(0) \Longrightarrow \beta(z) = \beta(0) \Longrightarrow z \in \beta_X$.

Definition 3.7. Let β be a fuzzy BOI-ideal of BOI-algebra X, the BOI-ideal $\beta_\theta = \{x \in X | \beta(x) \leq \theta\}$, for $\theta \in [0,1]$ is called a lower θ-level BOI-ideal of β.

Theorem3.8. Let β be a fuzzy subset of a BOI-algebra X. If β is a doubt fuzzy BOI-ideal of X, then for each $\theta \in [0,1], \theta \geq \beta(0)$, the lower θ-level cut β_θ is a BOI-ideal of X.

Proof. Let β be a doubt fuzzy BOI-ideal of X and let $\theta \in [0,1]$ with $\theta \geq \beta(0)$. Clearly $0 \in \beta_\theta$.

Let $x, y, z \in X$ be such that $(x * y) \in \beta_\theta$ and $((x * z) * y) \in \beta_\theta$.

Then $\beta(x * y) \leq \theta$, and $\beta((x * z) * y) \leq \theta \Rightarrow \beta(z) \leq \langle \beta(x * y)) \vee \beta((x * z) * y) \rangle \leq \theta \Rightarrow z \in \beta_\theta$.

Hence β_θ is a BOI-ideal of X.

Definition 3.9 (Image and Inverse Image). Let f be a mapping from the set X to the set Y. If β is a fuzzy subset of X, then the fuzzy subset A of Y defined by

$$\beta f^{-1}(y) = A(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \beta(x), & \text{if } f^{-1}(y) = \{x \in X, f(x) = y\} \neq \phi \\ 0 & \text{otherwise} \end{cases},$$

for all $y \in Y$ is called the image of β under f.

Similarly, if A is a fuzzy subset of Y, then the fuzzy subset defined by $A(f(x)) = \beta(x)$ for all $x \in X$, is said to be the inverse image of A under f.

Theorem3.10. The epimorphic image of a doubt fuzzy BOI-ideal is also a doubt fuzzy BOI-ideal.

Proof. Let $f : X \to Y$ be an epimorphism of BOI-algebras $(X; *, 0)$ and $(Y; *', 0')$. Consider that β is a doubt fuzzy BOI-ideal of X and μ is the image of β under f.

Let $y \in Y$. Then there exists $x \in X$ such that $f(x) = y$. Then

$\mu(y) = \mu(f(x)) = \beta(x) \geq \beta(0) = \mu(f(0)) = \mu(0')$.

Let $x', y', z' \in Y$. Then there exist $x, y, z \in X$. Such that $f(x) = x'$, $f(y) = y'$ and $f(z) = z'$. It follows that

$$\begin{aligned}\mu(z') = \mu(f(z)) = \beta(z) \leq & \left\langle \beta(x * y) \vee \beta((x * z) * y) \right\rangle \\ &= \left\langle \mu(f(x * y)) \vee \mu(f((x * z) * y)) \right\rangle \\ &= \left\langle \mu(f(x) *' f(y)) \vee \mu((f(x) *' f(z)) *' f(y)) \right\rangle \\ &= \mu(x' *' y') \vee \mu((x' *' z') *' y')\end{aligned}$$

Hence μ is a doubt fuzzy BOI-ideal of *Y*.

Theorem 3.11. The onto homomorphic inverse image of a doubt fuzzy BOI-ideal is also a doubt fuzzy BOI-ideal.

Proof. Let $f : X \to Y$ be an onto homomorphism of BOI-algebras $(X; *, 0)$, $(Y; *', 0')$. And β is a doubt fuzzy BOI-ideal of Y and μ is the inverse image of β under f.

By definition (3.9) we find that $\beta(f(x)) = \mu(x)$, for all $x \in X$, since β is a doubt fuzzy BOI-ideal of *Y, then* $\beta(0') \leq \beta(f(x))$, *for all* $x \in X$.

Since $\mu(0) = \beta(f(0)) = \beta(0') \leq \beta(f(x)) = \mu(x)$ then $\mu(0) \leq \mu(x)$.

For all $x, y, z \in X$ we have

$$\begin{aligned}\mu(z) = \beta(f(z)) \leq & \left\langle \beta(f(x) *' f(y)) \vee \beta((f(x) *' f(z)) *' f(y)) \right\rangle \\ &= \left\langle \beta(f(x * y)) \vee \beta(f((x * z) * y)) \right\rangle = \left\langle \mu(x * y) \vee \mu((x * z) * y) \right\rangle.\end{aligned}$$

Hence $\mu(z) = \beta(f(z)) = (\beta \circ f)(z)$ is a doubt fuzzy BOI-ideal of *X*.

The proof is complete.

4. Cartesian product of Doubt fuzzy BOI-ideal

Definition 4.1[3]. A fuzzy relation on any set S is a fuzzy subset $\mu : S \times S \to [0,1]$.

Definition 4.2 [3]. Let μ and β be the fuzzy subsets of a set S. The doubt Cartesian product $\mu \times \beta : X \times X \to [0,1]$ is defined by $(\mu \times \beta)(x, y) = \mu(x) \vee \beta(y)$, for all $x, y \in S$.

Definition 4.3 [3]. Let μ and β be fuzzy subsets of a set S. The Cartesian product of μ and β is defined by $(\mu \times \beta)(x, y) = \mu(x) \wedge \beta(y)$ for all $x, y \in S$.

Corollary 4.4[7]. Let $(X; *, 0)$ and $(Y; *, 0')$ be BOI-algebras, we define $*$ on $X \times Y$ by For every $(x_1, x_2), (y_1, y_2) \in X \times Y$ $(x_1, x_2) * (y_1, y_2) = (x_1 * y_1, x_2 * y_2)$ then $(X \times Y; *, (0, 0'))$ is a BOI-algebra.

Theorem 4.5. If μ and β are doubted fuzzy BOI-Ideals of BOI-algebras X, then $\mu \times \beta$ is a doubt fuzzy BOI-ideal of $(X \times X; *, (0, 0'))$.

Proof. Let $x, x' \in X \times X$. Then

$$(\mu \times \beta)(0, 0') = \mu(0) \vee \beta(0') \leq \mu(x) \vee \beta(x') = (\mu \times \beta)(x, x')$$

For any $(x, x'), (y, y') \in X \times X$ we have

$$\begin{aligned}(\mu \times \beta)(z, z') = \mu(z) \vee \beta(z') \\ &\leq \langle \mu(x * y) \vee \mu((x * z) * y) \rangle \vee \langle \beta(x' * y') \vee \beta((x' * z') * y') \rangle \\ &= \langle \mu(x * y) \vee \beta(x' * y') \rangle \vee \langle \mu((x * z) * y) \vee \beta((x' * z') * y') \rangle \\ &= (\mu \times \beta)((x, x') * (y, y')) \vee (\mu \times \beta)(((x, x') * (z, z')) * (y, y')).\end{aligned}$$

Hence $\mu \times \beta$ is a doubt fuzzy BOI-ideal of $(X \times X; *, (0, 0'))$.

Definition 4.6. Define $\nabla_\gamma(x, y) = \gamma(x) \vee \gamma(y)$ for all $x, y \in A$ is the strongest doubt fuzzy relation on the set A, while γ is a fuzzy subset of a set A.

Proposition 4.7. For a given doubt fuzzy subset γ of a BOI-algebra X, let ∇_γ be the strongest doubt fuzzy relation on X. If ∇_γ is a doubt fuzzy BOI-ideal of $(X \times X; *, (0,0))$, then $\gamma(0) \leq \gamma(x)$ for all $x \in X$.

Proof. Since ∇_γ is a doubt fuzzy BOI-ideal of $X \times X \implies \nabla_\gamma(0,0) \leq \nabla_\gamma(x, x)$. So that $\nabla_\gamma(0,0) = \gamma(0) \vee \gamma(0) \leq \gamma(x) \vee \gamma(x) = \nabla_\gamma(x, x) \implies \gamma(0) \leq \gamma(x)$.

Theorem 4.8. Let γ *and* ∇_γ be a doubt fuzzy subset of BOI-algebras X and the strongest doubt fuzzy relation on X consequently. If γ is a doubt fuzzy BOI-ideal of X then ∇_γ is a doubt fuzzy BOI-ideal of $(X \times X; *, (0, 0'))$.

Proof. Suppose that γ is a doubt fuzzy subset of a BOI-ideal X, and ∇_γ is the strongest doubt fuzzy relation on X. Then $\nabla_\gamma(0, 0') = \gamma(0) \vee \gamma(0') \leq \gamma(x) \vee \gamma(y) = \nabla_\gamma(x, y)$, for all $(x, y) \in X \times X$.

For all $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$, we get that

$$\nabla_{\gamma}(z_1, z_2) = \gamma(z_1) \vee \gamma(z_2) \le \left\langle \left\langle \gamma(x_1 * y_1) \vee \gamma((x_1 * z_1) * y_1) \right\rangle \vee \left\langle \gamma(x_2 * y_2) \vee \gamma((x_2 * z_2) * y_2) \right\rangle \right\rangle$$
$$= \left\langle \left\langle \gamma(x_1 * y_1) \vee \gamma(x_2 * y_2) \right\rangle \vee \left\langle \gamma((x_1 * z_1) * y_1) \vee \gamma((x_2 * z_2) * y_2) \right\rangle \right\rangle$$
$$= \left\langle \nabla_{\gamma}((x_1 * y_1), (x_2 * y_2)) \ \vee \ \nabla_{\gamma}(((x_1 * z_1) * y_1), ((x_2 * z_2) * y_2)) \right\rangle$$
$$= \left\langle \nabla_{\gamma}((x_1, x_2) * (y_1, y_2)) \vee \nabla_{\gamma}(((x_1, x_2) * (z_1, z_2)) * (y_1, y_2)) \right\rangle$$

Hence ∇_{γ} is a doubt fuzzy BOI-ideal of $X \times X$.

5. Conclusion and Future Research

To investigate the structure of an algebraic system, it is clear that BOI-ideal with special properties plays an important role. In the present paper, we have applied the concept of doubt fuzzy set to BOI-ideal of BOI-algebra and investigated some of their useful properties. In the future, these definitions and fundamental results can be applied to some different algebraic structures. There are many other aspects which should be explored and studied in the area of BOI-algebra such as doubt intuitionistic fuzzy BOI-ideal of BOI-algebra, intuitionistic fuzzy BOI-ideal of BOI-algebra, fuzzy soft BOI-ideal of BOI-algebra, bipolar fuzzy BOI- ideal of BOI-algebra, interval-valued fuzzy BOI-ideal of BOI-algebra, fuzzy derivations BOI-ideal of BOI-algebra, and interval-valued intuitionistic fuzzy BOI-ideal of BOI-algebra. It is our hope that this work would other foundations for further study of the theory of BOI-algebra.

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