

# The remarks on the four color problem

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## Abstract

The proof follows the Euler Identity for planar graphs. In this note we are proving the four color conjecture for planar graphs.

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## Keywords

Four color conjecture, planar graphs.

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## 1 Introduction

We will concern notoriously difficult, the four color problem for planar graphs.

A graph  $G$  is a planar graph if  $G$  can be drawn in the plane so that no two of its edges cross. Let  $G$  be a connected planar graph with  $n$ -vertices,  $m$ -edges and  $r$ -regions.

The four color conjecture, famous from 1860: the regions of every planar graph can be colored with four or fewer colors in such a way that every two regions sharing a common boundary boundary are colored differently [ 1 ].

For many years unsolved, the four color problem was attacked unsuccessfully by very famous mathematicians [ 1 ], [2]. Finally in 1976 two americans mathematicians: Kenneth Appel and Wolfgang Hake gave a proof. The proof is highly controversial because related on computer. .

In 1996 group of four Americans mathematicians: Neil Robertson, Daniel Sanders, Paul Seymour and Robin Thomas relied heavily on computer and used the same approach gave own proof.

Amazingly, each of the proofs required different number of the so called "unavoidable configurations". What does it mean it is unclear. The question remaining, are there real proofs of four colors conjecture?

## 2. Proof of the four color conjecture.

If  $G$  is a connected planar graph with  $n$ -vertices,  $m$ -edges and  $r$ -regions then

$$(1) \quad n - m + r = 2 \quad (\text{the Euler Identity for planar graphs [1]})$$

We rewrite (1) in the form

$$(2) \quad r = 2 - n + m.$$

We will ask about the minimum of (2) under following assumptions

$$(3) \quad n \geq 3$$

$$(4) \quad m \leq 6$$

$$(5) \quad m \leq 3n - 6$$

The assumption (3) means, we consider the graphs of minimum 3 vertices.

The assumption (4) means, we consider the planar graphs of maximum 6 edges. Though, the maximal planar graph contains 9 edges, the same numbers of edges contains non planar graph  $K_{3,3}$ .

The assumption (5) is one of the theorem of the planar graphs [1].

The system of inequalities (3)→(5) gives us as solution only one point (4,6). This means that  $r = 4$ .

## 3. Conclusion

Because theorem (5):  $m \leq 3n - 6$  is proven using the Euler Identity we see that the proof of four color theorem real is based on the Euler Identity.

## References

- [1] Arthur Benjamin, Gary Chartrand, Ping Zhung, The fascinating word of graph theory, Princeton University Press, 2015.
- [2] en. wikipedia.org/wiki/four color theorem