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Rogue wave solutions for an inhomogeneous fifth-order nonlinear Schrödinger equation from Heisenberg ferromagnetism

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Abstract:

In this paper, generalized Darboux transformation for an inhomogeneous fifth-order nonlinear Schrödinger equation from Heisenberg ferromagnetism are constructed according to which rouge wave solutions of the equation are obtained. Influences of equation parameter on the evolution of rogue waves are discussed. With the aid of Mathematica, some special solutions are graphically illustrated which could help to better understand the evolution of rogue waves.

Keywords: Fifth-order nonlinear Schrödinger equation; Generalized Darboux transformation; Rogue wave.

1 Introduction

The concept of rogue wave first appeared in studies of deep ocean waves [1-3] and gradually transfer to other fields such as optics [4-6], Bose-Einstein condensates [7-9], plasmas [10, 11], etc. Rogue waves are localized in both space and time, appear from nowhere and disappear without a trace [12].

Nonlinear Schrödinger (NLS) equations could describe a large number of important phenomena and dynamic processes in physics, chemistry, biology and computer science [13, 14]. Hence the explicit solutions of NLS equation play a vital role in practical applications. Searching exact solutions of NLS equation is one of the hot topics in nonlinear science.

To search rogue wave solutions of the NLS equation, there are two main approaches available: a method based on Wronskian determinants which could be seen in [15-18], and the generalized Darboux Transformation (gDT) method [19-22], which will be used in this paper to investigate an inhomogeneous fifth-order NLS equation from Heisenberg ferromagnetism.

Among the nonlinear integrable systems, the Heisenberg ferromagnetic equation [23] is a model describing the nonlinear dynamics of magnets. The model can be applied to spintronic devices such as magnetic-field sensors and high-density data storage [24-26]. In [26], the nonlinear spin dynamics of a site-dependent ferromagnet with inhomogeneous exchange interaction in the presence of relativistic Gilbert damping have been studied. The influences of inhomogeneity and damping on the evolution of energy of the magnetization are also demonstrated in [26]. Soliton solution of an inhomogeneous fifth-order nonlinear Schrödinger equation from Heisenberg ferromagnetism with variable coefficients have been given in [27]. In this paper, we consider the following inhomogeneous fifth-order NLS equation

$$iq_t - i\varepsilon q_{xxxxx} - 10i\varepsilon |q|^2 q_{xxx} - 20i\varepsilon q_x q^* q_{xx} - 30i\varepsilon |q|^4 q_x - 10i\varepsilon(|q_x|^2 q)_x + q_{xx} + 2q|q|^2 - iq_x = 0, \quad (1)$$

where $q = q(x,t)$ is is a complex function, x and t denote the spatial coordinate and scaled time, ε is a perturbation parameter, the asterisk represents the complex conjugate. Lax pair and classical Darboux Transformation (cDT) of Eq. (1) can be found in [23, 27].

cDT is a technique to iterate the soliton solutions [27-30], but it cannot be constructed at the same spectral parameter [22, 27, 31]. In order to overcome this difficulty, gDT has been proposed via the Taylor expansion and a limiting process [31]. So far, gDT has successfully been applied to construct the rogue wave solutions of various nonlinear evolution equations such as the Hirota and coupled Hirota equations [32, 33], the coupled nonlinear Schrödinger equations [34], the AB equation [35], the Kundu-Eckhaus equation [36], etc.

The main purpose of this paper is to apply gDT to investigate rogue wave solutions for the equation (1). In Section 2, Lax pair and gDT matrix of the equation are given. In Section 3, rogue wave solutions for the equation are derived via the gDT. Discussion and conclusion are given in the last section.

2 gDT for the fifth-order NLS equation (1)

The inhomogeneous fifth-order NLS equation (1) is the compatibility of the following Lax pair:

$$\Psi_t = U\Psi, \quad \Psi_x = V\Psi, \tag{2}$$

where $\Psi = (\psi_1, \psi_2)^T$ is a vector eigenfunction and T represents the transpose of a matrix. ψ_1 and ψ_2 are the functions of x and t. The matrices U and V are given by

$$U = \begin{pmatrix} -i\lambda & q(x,t) \\ -q^*(x,t) & i\lambda \end{pmatrix}, \quad V = \begin{pmatrix} V_{11} & V_{12} \\ -V_{12}^* & -V_{11} \end{pmatrix}, \tag{3}$$

where

$$\begin{aligned} V_{11} = \; & -16i\lambda^5\varepsilon + 8i\lambda^3\varepsilon |q|^2 + 4\lambda^2\varepsilon(qq_x^* - q_x q^*) - 2i\lambda^2 - 2i\lambda\varepsilon(qq_{xx}^* + q^* q_{xx} \\ & - |q_x|^2 + 3|q|^4) - i\lambda + i|q|^2 + \varepsilon(q^* q_{xxx} - qq_{xxx}^* + q_x q_{xx}^* - q_{xx} q_x^* \\ & + 6|q|^2 q^* q_x - 6|q|^2 q_x^* q), \end{aligned}$$

$$\begin{aligned} V_{12} = \; & 16i\lambda^4\varepsilon q + 8i\lambda^3\varepsilon q_x - 4\lambda^2\varepsilon(q_{xx} + 2|q|^2 q) - 2i\lambda\varepsilon(q_{xxx} + 6|q|^2 q_x^*) + 2\lambda q \\ & + iq_x + q + \varepsilon(q_{xxxx} + 8|q|^2 q_{xx} + 2q^2 q q_{xx}^* + 4|q_x|^2 q + 6q_x^2 q^* + 6|q|^4 q). \end{aligned}$$

and λ is a parameter independent of x and t. It is easy to verify that equation (1) can be reproduced from the compatibility condition $U_t - V_x + UV - VU = 0$.

In order to construct the cDT for Lax pair (2), we suppose that $q^{[0]}$ is a seed solution of the equation (1). Let $(\phi_{11}, \phi_{12})^T$ is a solution of the Lax pair (2) at $\lambda = \lambda_1, q = q^{[0]}$. It is obvious that $(-\phi_{12}^*, \phi_{11}^*)^T$ is also a solution of the Lax pair (2) with $\lambda = \lambda_1^*, q = q^{[0]}$. The cDT matrix $M^{[1]}$ can be constructed as [27, 37]

$$\begin{aligned} & M^{[1]} = \lambda I - S^{[1]}, \qquad S^{[1]} = H^{[1]}\Lambda(H^{[1]})^{-1}, \\ & H^{[1]} = \begin{pmatrix} \phi_{11} & -\phi_{12}^* \\ \phi_{12} & \phi_{11}^* \end{pmatrix}, \quad \Lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_1^* \end{pmatrix}, \end{aligned} \tag{4}$$

where I denotes the 2×2 identity matrix, $(H^{[1]})^{-1}$ is the inverse of $H^{[1]}$. The superscript $[i](i = 1,2,\ldots)$ denotes the i-th cDT of matrix and function. With the help of the symbolic computation [38], it can be discovered that the form of Lax pair (2) keeps unchanged under the operation of matrix $M^{[1]}$. Through computation, we can acquire $q^{[1]}$ as

$$q^{[1]} = q^{[0]} - \frac{2(\lambda_1 - \lambda_1^*)\phi_{12}\phi_{11}^*}{\phi_{11}\phi_{11}^* + \phi_{12}\phi_{12}^*}. \tag{5}$$

Based on the above cDT, the gDT for the equation (1) can be constructed. Supposing $\Phi(\lambda_1 + \delta)$ is a special solution of the Lax pair (2) with $\lambda = \lambda_1 + \delta, q = q^{[0]}$, where δ is a small parameter. Expanding $\Phi(\lambda_1 + \delta)$ in the Taylor series at $\delta = 0$, we have

$$\Phi(\lambda_1 + \delta) = \Phi_0 + \Phi_1\delta + \Phi_2\delta^2 + \cdots + \Phi_k\delta^k + \cdots,$$

where $\Phi_k = \dfrac{1}{k!}\dfrac{\partial^k \Phi(\lambda_1)}{\partial\lambda^k}|_{\lambda=\lambda_1} (k = 0,1,2,\ldots)$. It is obvious that $\Phi_0 = (\phi_{11}, \phi_{12})^T$ is a solution of the Lax pair (2) with $\lambda = \lambda_1, q = q^{[0]}$.

The first-order gDT can be constructed via Eqs. (4) and (5) with the above procedure for constructing cDT.

It can be shown that $M^{[1]}|_{\lambda=\lambda_1+\delta}\,\Phi(\lambda_1+\delta)$ is a solution of the Lax pair (4) with $\lambda = \lambda_1+\delta, q = q^{[1]}$. With the help of the identity $M^{[1]}|_{\lambda=\lambda_1}\,\Phi_0 = 0$ [31], the limit process

$$\lim_{\delta\to 0}\frac{M^{[1]}|_{\lambda=\lambda_1+\delta}\,\Phi(\lambda_1+\delta)}{\delta} = \lim_{\delta\to 0}\frac{(\delta I + M^{[1]}|_{\lambda=\lambda_1})\Phi(\lambda_1+\delta)}{\delta}$$
$$= \Phi_0 + M^{[1]}|_{\lambda=\lambda_1}\,\Phi_1 = (\phi_{21},\phi_{22})^T, \tag{6}$$

provides a nontrivial solution of the Lax pair (2) with $\lambda = \lambda_1, q = q^{[1]}$, which can be used to obtain the second-order gDT matrix $M^{[2]}$ as

$$M^{[2]} = \begin{pmatrix}\lambda & 0\\ 0 & \lambda\end{pmatrix} - \begin{pmatrix}\phi_{21} & -\phi_{22}^*\\ \phi_{22} & \phi_{21}^*\end{pmatrix}\begin{pmatrix}\lambda_1 & 0\\ 0 & \lambda_1^*\end{pmatrix}\begin{pmatrix}\phi_{21} & -\phi_{22}^*\\ \phi_{22} & \phi_{21}^*\end{pmatrix}^{-1}. \tag{7}$$

The second-order solutions for the equation (1) can be constructed as

$$q^{[2]} = q^{[1]} - \frac{2(\lambda_1-\lambda_1^*)\phi_{22}\phi_{21}^*}{\phi_{21}\phi_{21}^*+\phi_{22}\phi_{22}^*}. \tag{8}$$

Similarly, with the identities $M^{[1]}|_{\lambda=\lambda_1}\,\Phi_0 = 0, M^{[2]}|_{\lambda=\lambda_1}\,(\Phi_0 + M^{[1]}|_{\lambda=\lambda_1}\,\Phi_1) = 0$ [31], the limit process

$$\lim_{\delta\to 0}\frac{(M^{[2]}M^{[1]})|_{\lambda=\lambda_1+\delta}\,\Phi(\lambda_1+\delta)}{\delta^2} = \lim_{\delta\to 0}\frac{(\delta\ \ I + M^{[2]}|_{\lambda=\lambda_1})(\delta I + M^{[1]}|_{\lambda=\lambda_1})\Phi(\lambda_1+\delta)}{\delta^2}$$
$$= \Phi_0 + (M^{[2]}|_{\lambda=\lambda_1} + M^{[1]}|_{\lambda=\lambda_1})\Phi_1 + M^{[2]}|_{\lambda=\lambda_1}\,M^{[1]}|_{\lambda=\lambda_1}\,\Phi_2 = (\phi_{31},\phi_{32})^T, \tag{9}$$

gives a nontrivial solution of the Lax pair (2) with $\lambda = \lambda_1, q = q^{[2]}$, which can be used to construct the third-order gDT matrix $M^{[3]}$, i.e.,

$$M^{[3]} = \begin{pmatrix}\lambda & 0\\ 0 & \lambda\end{pmatrix} - \begin{pmatrix}\phi_{31} & -\phi_{32}^*\\ \phi_{32} & \phi_{31}^*\end{pmatrix}\begin{pmatrix}\lambda_1 & 0\\ 0 & \lambda_1^*\end{pmatrix}\begin{pmatrix}\phi_{31} & -\phi_{32}^*\\ \phi_{32} & \phi_{31}^*\end{pmatrix}^{-1}, \tag{10}$$

The third-order solutions for equation (1) can be given by

$$q^{[3]} = q^{[2]} - \frac{2(\lambda_1-\lambda_1^*)\phi_{32}\phi_{31}^*}{\phi_{31}\phi_{31}^*+\phi_{32}\phi_{32}^*}. \tag{11}$$

In a similar way, the N th-order gDT can be constructed following the above process.

3 Rogue wave solutions of the fifth-order NLS equation (1)

With the aid of gDTs obtained in Section 2, we can construct the rogue wave solutions for equation (1). Let us start with the seed solution

$$q^{[0]} = ie^{2it} \tag{12}$$

of equation (1). The corresponding solution for Lax pair (2) at $\lambda = -is$ is

$$\Phi = \begin{pmatrix} i(a_1e^{\eta} - a_2e^{-\eta})e^{it} \\ (a_2e^{\eta} - a_1e^{-\eta})e^{-it}\end{pmatrix}, \tag{13}$$

where

$$a1=\frac{\sqrt{s-\sqrt{s^2-1}}}{\sqrt{s^2-1}},\quad a2=\frac{\sqrt{s+\sqrt{s^2-1}}}{\sqrt{s^2-1}},\quad \eta=\mu(x+kt),$$
$$\mu=\sqrt{-1+s^2},\qquad k=6\varepsilon+16\varepsilon s^4+8\varepsilon s^2-2is+1, \tag{14}$$

Taking $s=1+f^2$, the vector function $\Phi(f)$ can be expanded as a Taylor series at $f=0$, namely,

$$\Phi=\Phi_0+\Phi_1\varepsilon^2+\Phi_2\varepsilon^4+\cdots. \tag{15}$$

Φ_0 can be expressed as

$$\Phi_0=\begin{pmatrix} ie^{it}((60\varepsilon+(2-4i))t+2x-1) \\ \\ e^{-it}((60\varepsilon+(2-4i))t+2x+1)\end{pmatrix}, \tag{16}$$

and $\Phi_1=(\varphi_{21},\varphi_{22})^T$, $\Phi_2=(\varphi_{31},\varphi_{32})^T$ are listed in Appendix A.

According to Eqs. $(4)-(11)$, we can obtain the first- and second-order rogue wave solutions for system (1). The first-order rogue wave solution is as follows

$$q^{[1]}=\frac{ie^{2it}\left(3600\varepsilon^2t^2+240\varepsilon t^2+20t^2+240\varepsilon tx+8tx-16it+4x^2-3\right)}{3600\varepsilon^2t^2+240\varepsilon t^2+20t^2+240\varepsilon tx+8tx+4x^2+1}. \tag{17}$$

The second-order rouge wave solution is presented in Appendix B.

4 Discussion and conclusion

Based on the rogue wave solutions acquired above, and with the aid of Mathematica, we can investigate the properties of rogue waves for equation (1). The first- and second-order rogue waves are displayed in Fig.1.

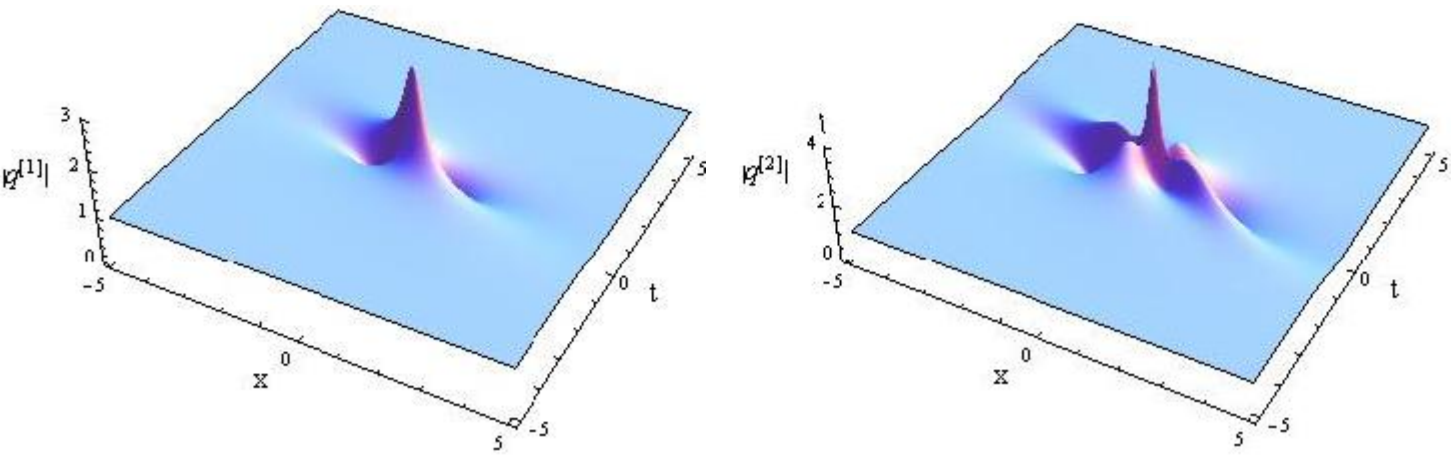
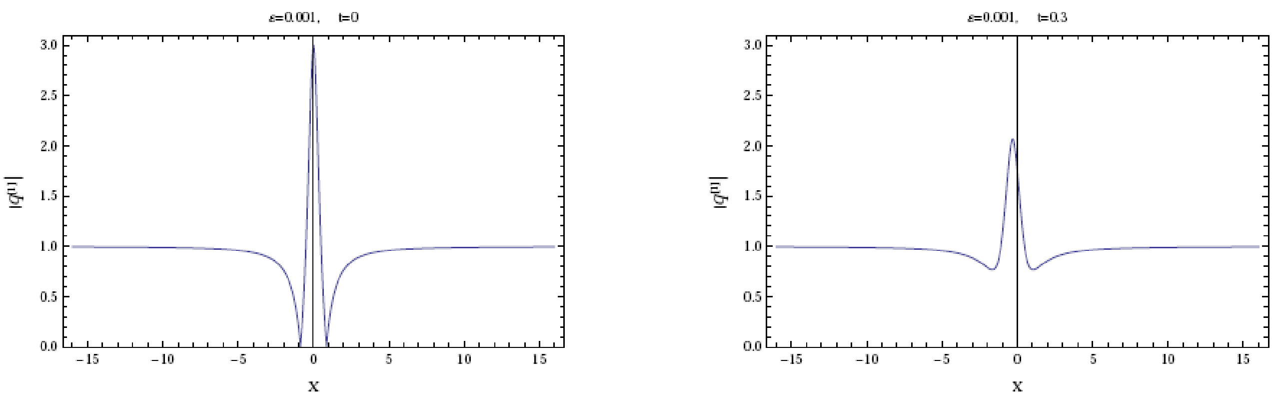


Fig.1. Plot of $|q^{[1]}|,|q^{[2]}|$ with $\varepsilon=0.0001$

Fig.2. and Fig.3. show that parameter ε affects the travelling speed of the first- and second-order rogue waves. The rogue waves with bigger ε travel faster.



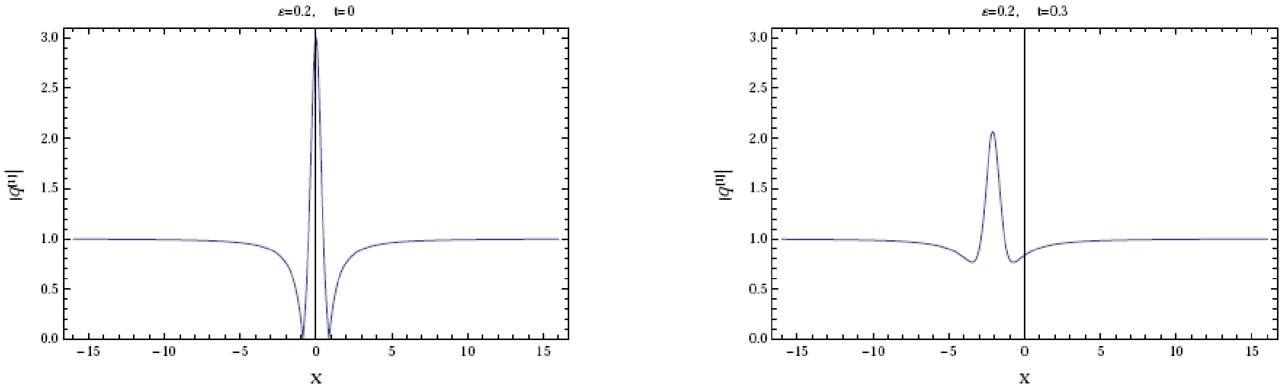


Fig.2. Effect of ε on the travelling of $|q^{[1]}|$.

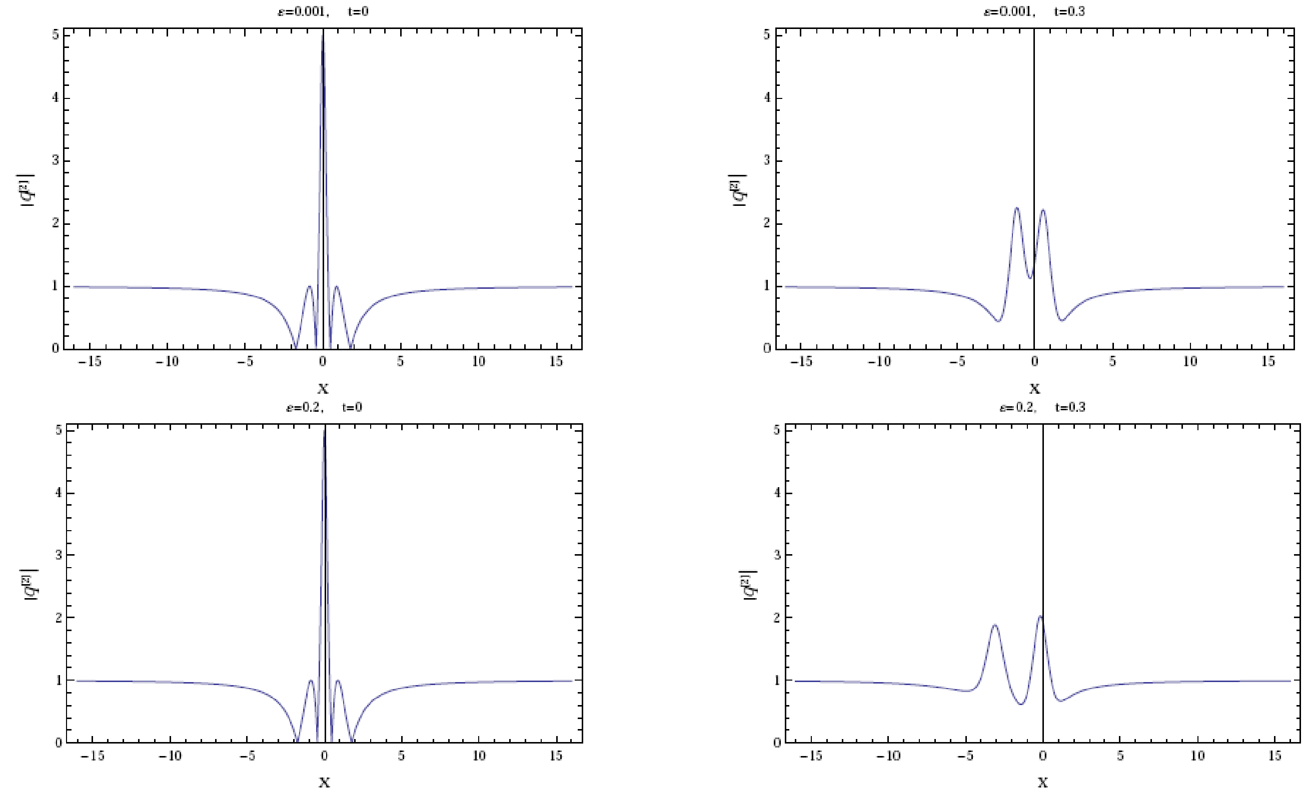
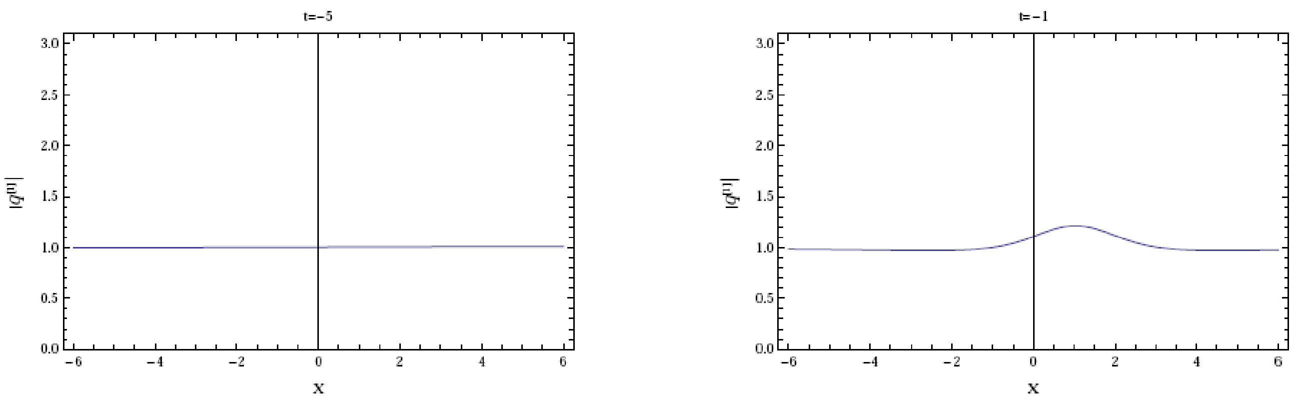


Fig.3. Effect of ε on the travelling of $|q^{[2]}|$.

Fig.4. shows the evolution of first-order rogue wave for system (1) with $\varepsilon = 0.001$.



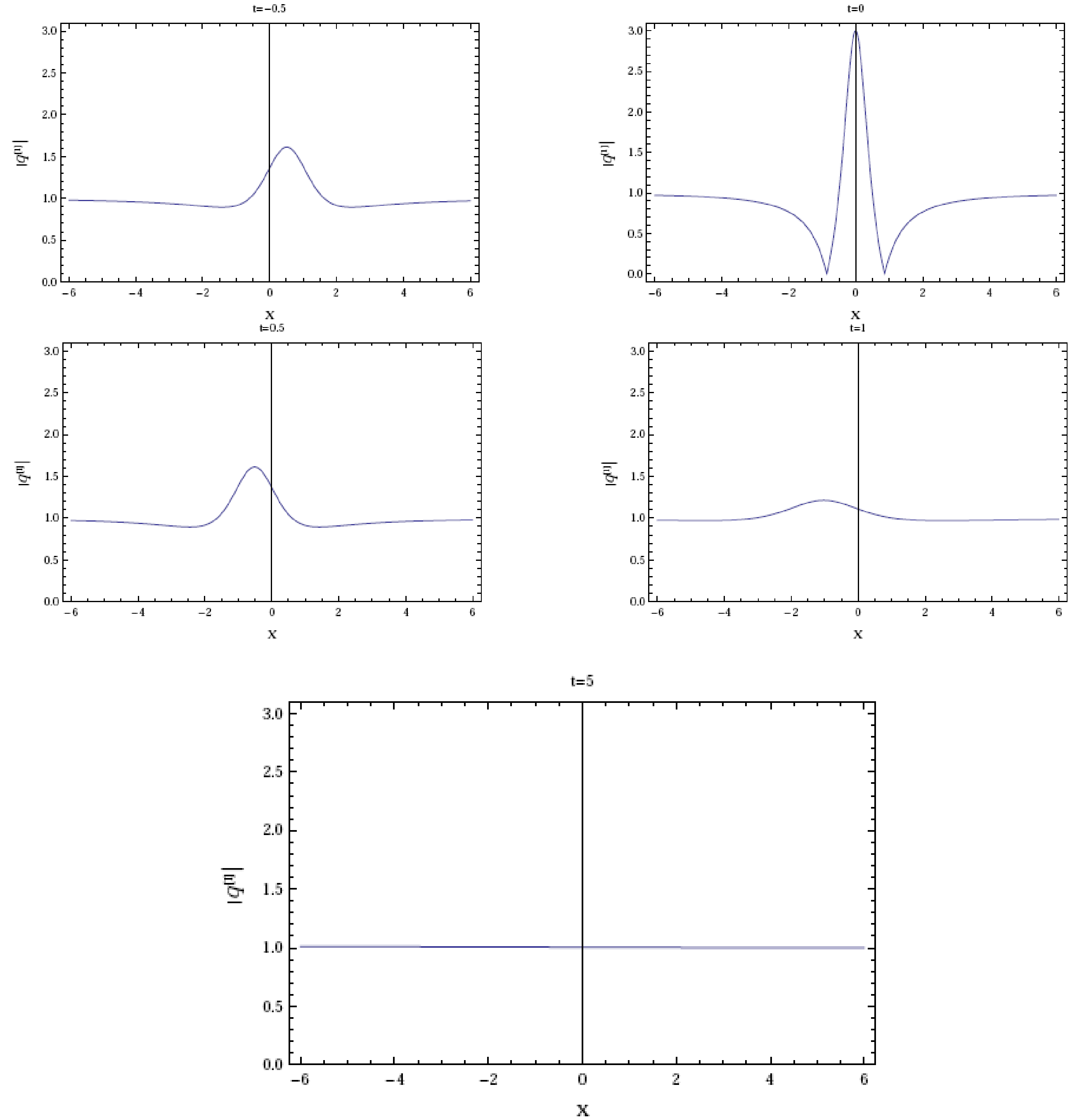
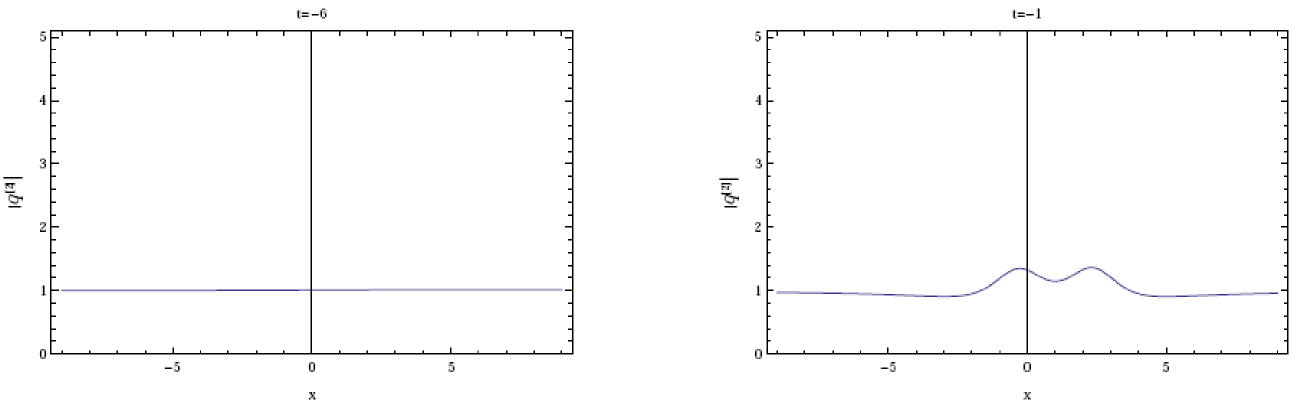


Fig.4. Evolution of the first-order rogue wave.

Fig.5. shows the evolution of second-order rogue wave for system (1) with $\varepsilon = 0.001$.



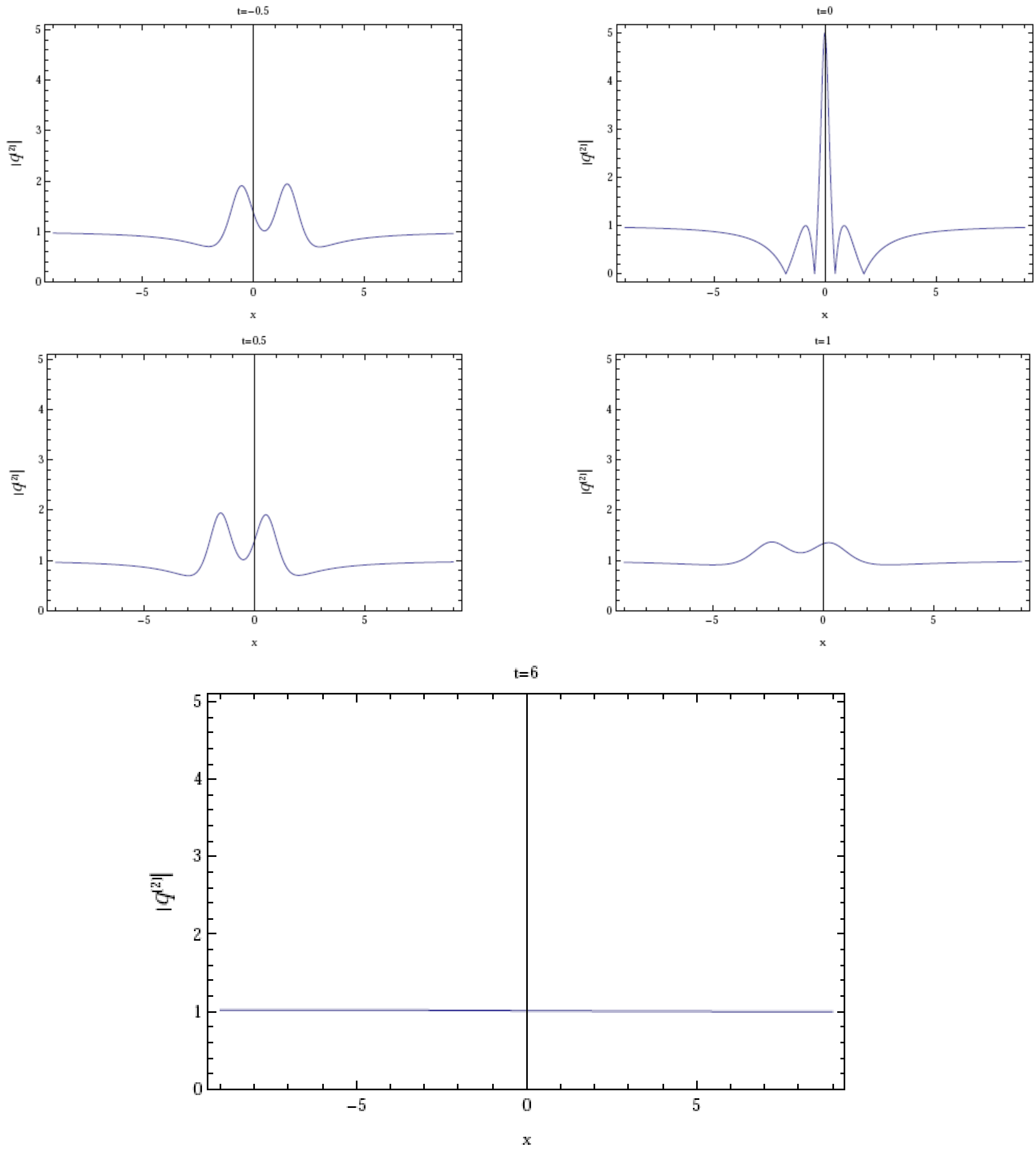


Fig.5. Evolution of the second-order rogue wave.

Fig.4 and Fig.5 show that the rogue waves of equation (1) have the typical evolution property of "appear from nowhere and disappear without a trace" [12].

In summary, we have investigated an inhomogeneous fifth-order nonlinear Schrödinger equation from Heisenberg ferromagnetism. Starting from the Lax pair (2), we obtained gDTs (4)−(11) for equation (1). The first- and second-order rogue wave solutions for equation (1) were then derived via the gDTs. Properties of the rogue waves have been graphically analyzed and the influences of equation parameter ε on the propagation of first and second-order rogue waves were discussed. The results obtained in this paper will help to better study the rogue waves in Heisenberg ferromagnetism.

Appendix A

The expressions for $\Phi_1 = (\varphi_{21}, \varphi_{22})^T$:

$$\varphi_{21} = \frac{e^{it}}{12}(216000i\varepsilon^3t^3 + (43200 + 21600i)\varepsilon^2t^3 + (2880 - 2160i)\varepsilon t^3 - (16 + 88i)t^3 - 10800i\varepsilon^2t^2 - (1440+720i)\varepsilon t^2 + 21600i\varepsilon^2t^2x + (2880+1440i)\varepsilon t^2x + (96-72i)t^2x - (48-36i)t^2 + 2100i\varepsilon t + 720i\varepsilon tx^2 + (48+24i)tx^2 - 720i\varepsilon tx - (48+24i)tx + (60+6i)t + 8ix^3 - 12ix^2 + 6ix + 3i),$$

$$\varphi_{22} = \frac{e^{-it}}{12}(216000\varepsilon^3t^3 + (21600 - 43200i)\varepsilon^2t^3 - (2160 + 2880i)\varepsilon t^3 - (88 - 16i)t^3 + 10800\varepsilon^2t^2 + (720 - 1440i)\varepsilon t^2 + 21600\varepsilon^2t^2x + (1440 - 2880i)\varepsilon t^2x - (72 + 96i)t^2x - (36 + 48i)t^2 + 2100\varepsilon t + 720\varepsilon tx^2 + (24-48i)tx^2 + 720\varepsilon tx + (24-48i)tx + (6-60i)t + 8x^3 + 12x^2 + 6x - 3).$$

The expressions for $\Phi_2 = (\varphi_{31}, \varphi_{32})^T$:

$$\varphi_{31} = \frac{e^{it}}{480}(777600000i\varepsilon^5t^5 + (259200000+129600000i)\varepsilon^4t^5 + (34560000-25920000i)\varepsilon^3t^5 - (576000 + 3168000i)\varepsilon^2t^5 - (115200 + 33600i)\varepsilon t^5 - (1216 - 1312i)t^5 - 64800000i\varepsilon^4t^4 - (17280000+8640000i)\varepsilon^3t^4 - (1728000-1296000i)\varepsilon^2t^4 + (19200+105600i)\varepsilon t^4 + 129600000i\varepsilon^4t^4x + (34560000+17280000i)\varepsilon^3t^4x + (3456000-2592000i)\varepsilon^2t^4x - (38400+211200i)\varepsilon t^4x - (3840 + 1120i)t^4x + (1920 + 560i)t^4 + 75600000i\varepsilon^3t^3 + (12240000 + 5256000i)\varepsilon^2t^3 + (508800 - 525600i)\varepsilon t^3 + 8640000i\varepsilon^3t^3x^2 + (1728000+864000i)\varepsilon^2t^3x^2 + (115200-86400i)\varepsilon t^3x^2 - (640 + 3520i)t^3x^2 - 8640000i\varepsilon^3t^3x - (1728000 + 864000i)\varepsilon^2t^3x - (115200 - 86400i)\varepsilon t^3x + (640 + 3520i)t^3x - (6240 + 10320i)t^3 - 2412000i\varepsilon^2t^2 - (225600 + 84000i)\varepsilon t^2 + 288000i\varepsilon^2t^2x^3 + (38400+19200i)\varepsilon t^2x^3 + (1280-960i)t^2x^3 - 432000i\varepsilon^2t^2x^2 - (57600+28800i)\varepsilon t^2x^2 - (1920 - 1440i)t^2x^2 + 5256000i\varepsilon^2t^2x + (508800+196800i)\varepsilon t^2x + (6720-9840i)t^2x - (2400-4200i)t^2 + 118140i\varepsilon t + 4800i\varepsilon tx^4 + (320+160i)tx^4 - 9600i\varepsilon tx^3 - (640+320i)tx^3 + 98400i\varepsilon tx^2 + (3360 + 720i)tx^2 - 84000i\varepsilon tx - (2400+240i)tx + (420-30i)t + 32ix^5 - 80ix^4 + 240ix^3 - 120ix^2 - 30ix - 45i),$$

$$\varphi_{32} = \frac{e^{-it}}{480}(777600000\varepsilon^5t^5+(129600000-259200000i)\varepsilon^4t^5-(25920000+34560000i)\varepsilon^3t^5-(3168000-576000i)\varepsilon^2t^5-(33600-115200i)\varepsilon t^5+(1312+1216i)t^5+64800000\varepsilon^4t^4+(8640000-17280000i)\varepsilon^3t^4-(1296000+1728000i)\varepsilon^2t^4-(105600-19200i)\varepsilon t^4+129600000\varepsilon^4t^4x+(17280000-34560000i)\varepsilon^3t^4x-(2592000+3456000i)\varepsilon^2t^4x-(211200-38400i)\varepsilon t^4x-(1120-3840i)t^4x-(560-1920i)t^4+75600000\varepsilon^3t^3+(5256000-12240000i)\varepsilon^2t^3-(525600+508800i)\varepsilon t^3+8640000\varepsilon^3t^3x^2+(864000-1728000i)\varepsilon^2t^3x^2-(86400+115200i)\varepsilon t^3x^2-(3520-640i)t^3x^2+8640000\varepsilon^3t^3x+(864000-1728000i)\varepsilon^2t^3x-(86400+115200i)\varepsilon t^3x-(3520-640i)t^3x-(10320-6240i)t^3+2412000\varepsilon^2t^2+(84000-225600i)\varepsilon t^2+288000\varepsilon^2t^2x^3+(19200-38400i)\varepsilon t^2x^3-(960+1280i)t^2x^3+432000\varepsilon^2t^2x^2+(28800-57600i)\varepsilon t^2x^2-(1440+1920i)t^2x^2+5256000\varepsilon^2t^2x+(196800-508800i)\varepsilon t^2x-(9840+6720i)t^2x-(4200+2400i)t^2+118140\varepsilon t+4800\varepsilon tx^4+(160-320i)tx^4+9600\varepsilon tx^3+(320-640i)tx^3+98400\varepsilon tx^2+(720-3360i)tx^2+84000\varepsilon tx+(240-2400i)tx-(30+420i)t+32x^5+80x^4+240x^3+120x^2-30x+45).$$

Appendix B

The Second-order rogue wave solutions are $q^{[2]} = \frac{s_1^{[2]}}{s_2^{[2]}}$, where

$$s_1^{[2]} = -ie^{-2it}(46656000000\varepsilon^6t^6+9331200000\varepsilon^5t^6+1399680000\varepsilon^4t^6+117504000\varepsilon^3t^6+7776000\varepsilon^2t^6+288000\varepsilon t^6+8000t^6+622080000i\varepsilon^4t^5+82944000i\varepsilon^3t^5+9676800i\varepsilon^2t^5+460800i\varepsilon t^5+9331200000\varepsilon^5t^5x+1555200000\varepsilon^4t^5x+186624000\varepsilon^3t^5x+11750400\varepsilon^2t^5x+518400\varepsilon t^5x+9600t^5x+19200it^5-531360000\varepsilon^4t^4-57024000\varepsilon^3t^4-1814400\varepsilon^2t^4-193920\varepsilon t^4+777600000\varepsilon^4t^4x^2+103680000\varepsilon^3t^4x^2+9331200\varepsilon^2t^4x^2+391680\varepsilon t^4x^2+8640t^4x^2+82944000i\varepsilon^3t^4x+8294400i\varepsilon^2t^4x+645120i\varepsilon t^4x+15360it^4x-14352t^4+4492800i\varepsilon^2t^3+115200i\varepsilon t^3+34560000\varepsilon^3t^3x^3+3456000\varepsilon^2t^3x^3+207360\varepsilon t^3x^3+4352t^3x^3+4147200i\varepsilon^2t^3x^2+276480i\varepsilon t^3x^2+10752it^3x^2-57024000\varepsilon^3t^3x-4320000\varepsilon^2t^3x-259200\varepsilon t^3x-12096t^3x+384it^3-277200\varepsilon^2t^2-45360\varepsilon t^2+864000\varepsilon^2t^2x^4+57600\varepsilon t^2x^4+1728t^2x^4+92160i\varepsilon t^2x^3+3072it^2x^3-2160000\varepsilon^2t^2x^2-97920\varepsilon t^2x^2-6624t^2x^2+115200i\varepsilon t^2x-2304it^2x-2052t^2+11520\varepsilon tx^5+384tx^5+768itx^4-32640\varepsilon tx^3-576tx^3-1152itx^2-45360\varepsilon tx-360tx-720it+64x^6-144x^4-180x^2+45),$$

$$s_2^{[2]} = 46656000000\varepsilon^6t^6+9331200000\varepsilon^5t^6+1399680000\varepsilon^4t^6+117504000\varepsilon^3t^6+7776000\varepsilon^2t^6+288000\varepsilon t^6+8000t^6+9331200000\varepsilon^5t^5x+1555200000\varepsilon^4t^5x+186624000\varepsilon^3t^5x+11750400\varepsilon^2t^5x+518400\varepsilon t^5x+9600t^5x-375840000\varepsilon^4t^4-36288000\varepsilon^3t^4+3369600\varepsilon^2t^4+105600\varepsilon t^4+777600000\varepsilon^4t^4x^2+103680000\varepsilon^3t^4x^2+9331200\varepsilon^2t^4x^2+391680\varepsilon t^4x^2+8640t^4x^2+5808t^4+34560000\varepsilon^3t^3x^3+3456000\varepsilon^2t^3x^3+207360\varepsilon t^3x^3+4352t^3x^3-36288000\varepsilon^3t^3x-2246400\varepsilon^2t^3x+86400\varepsilon t^3x-2112t^3x+1364400\varepsilon^2t^2+18000\varepsilon t^2+864000\varepsilon^2t^2x^4+57600\varepsilon t^2x^4+1728t^2x^4-1123200\varepsilon^2t^2x^2-28800\varepsilon t^2x^2-864t^2x^2+1692t^2+11520\varepsilon tx^5+384tx^5-9600\varepsilon tx^3+192tx^3+18000\varepsilon tx+216tx+64x^6+48x^4+108x^2+9.$$

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