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On $I\alpha$- Open Set in Ideal Topological Spaces

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Abstract

The aim of this paper is to study separation axioms and compactness of Iα- open set in ideal topological spaces, which was introduced by M.E. Abd El-Monsef, etc [1].

Keywords: $I\alpha$- open set; $I\alpha$- $T_\circ$- space; $I\alpha$- T_1- space; $I\alpha$- T_2- space; $I\alpha$- R- space; $I\alpha$- N- space and $I\alpha$- compact space.

1. Introduction

The subject of ideals in topological spaces has been studied by Kuratowski [3] and Vaidyanathaswamy [7]. An ideal I on a set X is a nonempty collection of subsets of X which satisfies: (1) $A \in I$ and $B \subset A$ implies $B \in I$ and (2) $A \in I, B \in A$ implies $A \cup B \in I$.

Given a topological space (X, τ) with ideal I on X and if $P(X)$ is the set of all subsets of X. A set operator $(\ \ )^*: P(X) \rightarrow P(X)$, called a local function [3] of A with respect to τ and I, is defined as follows: for $A \subset X$, $A^*(I, \tau) = \{x \in X: A \cap U \notin I \text{ for every } U \in \tau(x)\}$, where $\tau(x) = \{U \in \tau: x \in U\}$. Kuratowski closure operator $cl^*(\ \ )$ for the topology $\tau^*(I, \tau)$, called the $\star$ -topology and finer than τ, is defined by $cl^*(A) = A \cup A^*(I, \tau)$ [6]. When there is no chance for confusion we will simply write A^* for $A^*(I, \tau)$ and τ^* for $\tau^*(I, \tau)$. If I is an ideal on X then (X, τ, I) is called an ideal topological space.

In an ideal topological space (X, τ, I), if $A \subset X$ then $int^*(A)$ will denote the interior of A in (X, τ^*). The closed subsets of X in (X, τ^*) are called $*$- closed sets. A subset A of an ideal topological space (X, τ, I) is $*$- closed if and only if $A^* \subset A$ [2].

For any ideal topological space (X, τ, I), the collection $\{V - J: \ V \in \tau \ and \ J \in I\}$ is a basis for τ^*[2]. The elements of $\tau^*$are called $*$- open sets. A subset A of an ideal topological space (X, τ, I) is said to be $*$- dense set if $cl^*(A) = X$. It is clear that, in an ideal topological space (X, τ, I), if $A \subset B \subset X$ then $A^* \subset B^*$ and so $cl^*(A) \subset cl^*(B)$.

Recall that, if (X,τ,I) is an ideal topological space and A is a subset of X then (A,τ_A,I_A), where τ_A is the relative topology on A and $I_A=\{A\ \cap J: J\in I\}$, is an ideal topological subspace.

Given a topological space (X,τ). A subset A of a space X is said to be α- open set if $A\subset int\left(cl\left(int(A)\right)\right)$. The family of all α- open subsets of a space (X,τ) forms a topology on X, called the α- topology on X and denoted by τ_α. It is finer than τ. If every nowhere dense set in a space (X,τ) is closed then $\tau_\alpha=\tau$ [5].

The concept of a set operator $(\quad)^{\alpha*}: P(X)\to P(X)$ was introduced by A. A. Nasef [4] in 1992 which is called an α- local function of I with respect to τ. It was defined as follows: for $A\subset X, A^{\alpha*}(I,\tau)=\{x\in X: A\cap U\notin I\ for\ every\ U\in\tau_\alpha(x)\}$
where $\tau_\alpha(x)=\{U\in\tau_\alpha: x\in U\}$. When there is no chance for confusion we will simply write $A^{\alpha*}$ for $A^{\alpha*}(I,\tau)$. An α^*- closure operator, denoted by $cl^{\alpha*}(\quad)$, for a topology $\tau^{\alpha*}(I,\tau)$ which is called the $*$-α -topology, finer than τ. It is defined as follows: $cl^{\alpha*}(A)(I,\tau)=A\cup A^{\alpha*}(I,\tau)$. When there is no ambiguity we will simply write $cl^{\alpha*}(A)$ for $cl^{\alpha*}(A)(I,\tau)$. A basis $\mathfrak{B}(I,\tau)$ for $\tau^{\alpha*}$ is described as follows: $\mathfrak{B}(I,\tau)=\{V-J:\ V\in\tau_\alpha\ and\ J\in I\}$. We will denote by $int^{\alpha*}(A)$ and $cl^{\alpha*}(A)$ the interior and closure of $A\subset\ (X,\tau,I)$ with respect to $\tau^{\alpha*}$. The elements of $\tau^{\alpha*}$ are called $\tau^{\alpha*}$- open sets. Closed subsets of X in $(X,\tau^{\alpha*})$ are called $\tau^{\alpha*}$- closed sets. A subset A of an ideal topological space (X,τ,I) is $\tau^{\alpha*}$- closed (respectively, $\tau^{\alpha*}$- dense) if and only if $A^{\alpha*}\subset A$ (respectively, $cl^{\alpha*}(A)=X$). In an ideal topological space (X,τ,I) if $A\subset B\subset X$ then $A^{\alpha*}\subset B^{\alpha*}$ and $cl^{\alpha*}(A)\subset cl^{\alpha*}(B)$. So $A^{\alpha*}\subset A^*$ and $cl^{\alpha*}(A)\subset cl^*(A)$.

Given a topological space (X,τ). A subset A of X is said to be $I\alpha$- open set if $A\subset int\left(cl^{\alpha*}\left(int(A)\right)\right)$. The family of all $I\alpha$- open subsets of a space (X,τ) is denoted by $I\alpha O(X)$ [1]. Consider the ideal topological spaces (X,τ,I) and (Y,σ,J) and define a function $f:(X,\tau,I)\longrightarrow(Y,\sigma,J)$ such that f is $I\alpha$-irresolute homeomorphism, i.e…, then if the ideal topological space (X,τ,I) has any property P and the ideal topological space (Y,σ,J) has the same property P then P is called $I\alpha$- topological property. A property $P$of an ideal topological space X is said to be $I\alpha$- hereditary property if and only if every $I\alpha$- subspace of X also possesses property P.

In the following subjects we need to remember some concepts introduced by Radwan in 2015 [6]. An $I\alpha$-boundary set is defined as: Let (X,τ,I) be an ideal topological space and $A\subseteq X$. $x\ \in X$ is said to be an $I\alpha$-boundary point of A if for every $I\alpha$- open neighborhood set for x satisfies that the intersection with A and A^c is nonempty set. The set of all $I\alpha$- boundary points of A is called $I\alpha$-boundary set of A and simply is denoted by $I\alpha$- $b(A)$. Let (X,τ,I) be an ideal topological space and $A\subseteq X$. Then $I\alpha$- closure set of A is defined by the union of A and $I\alpha$-derived set of A and simply is denoted by $I\alpha$- $cl(A)$. Let $f:(X,\tau,I)\to(Y,\sigma,J)$ be a function. f is said to be $I\alpha$- irresolute function if the inverse image of every $J\alpha$- open set in Y is $I\alpha$- open set in X. $I\alpha$- topological property is a new property which defined as Consider the ideal topological spaces (X,τ,I) and (Y,σ,J) such that $f:(X,\tau,I)\longrightarrow(Y,\sigma,J)$ is $I\alpha$- irresolute function. Then if X satisfies any property P so did Y then this property is called $I\alpha$- topological property.

2. SEPARATION AXIMOS IN Iα- OPEN SETS

In this section, we will study some properties of different types of separation axioms on the level of ideal topological spaces using Iα- open sets.

Definition 2.1 Let (X,τ,I) be an ideal topological space then it is called $I\alpha$- $T_\circ$- space if for any two different elements in X there exist an $I\alpha$- open set in X containing one element of them but does not contain the other element.

Proposition 2.2 (X,τ,I) is an $I\alpha$- $T_\circ$- space if and only if $I\alpha$- $cl(x) \neq I\alpha$-$cl(y)$ for any $x,y \in X$ such that $x \neq y$.

Remark 2.3 The $I\alpha$- $T_\circ$- property is not hereditary property.

Example 2.4 Consider the ideal topological space (X,τ,I) such that $X=\{x,y,z\}, \tau=\mathcal{P}(X), I=\{\emptyset,\{y\}\}$ then $I\alpha O(X)=\mathcal{P}(X)$. It is clear that X is $I\alpha$-$T_\circ$- space but if we take $w=\{x\}\subseteq X$, then (w,τ_w,I_w) is not $I\alpha$-$T_\circ$- space for $I\alpha O(w)=\{w,\emptyset\}$.

Corollary 2.5 The $I\alpha$- $T_\circ$- property is $I\alpha$- hereditary property.

Remark 2.6 The continuous image of $I\alpha$- $T_\circ$- space is not necessary to be $J\alpha$- $T_\circ$ as the following example shows.

Example 2.7 Consider the ideal topological spaces (X,τ,I) and (X,τ,J) such that $X=\{a,b,c,d\}, \tau=\{X,\emptyset,\{a\},\{a,b\}\}, I=\mathcal{P}(X)$ and $J=\{\emptyset\}$.
Then $I\alpha O(X)=\tau$ and $J\alpha O(X)=\{X,\emptyset,\{a\},\{a,b\},\{a,c\},\{a,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\}\}$.

Define a function $f:(X,\tau,I)\longrightarrow(Y,\sigma,J)$ such that $f(a)=f(b)=f(c)=f(d)=\{a\}$. Then $f^{-1}(X)=X\in I\alpha O(X), f^{-1}(\emptyset)=\emptyset\in I\alpha O(X)$ and

$f^{-1}\{a\}=f^{-1}\{a,b\}=f^{-1}\{a,c\}=f^{-1}\{a,c,d\}=f^{-1}\{a,b,d\}=X\in I\alpha O(X)$. Thus f is $I\alpha$-continuous function and it is clear that (X,τ,J) is $I\alpha$-$T_\circ$- space but (X,τ,I) is not $I\alpha$- $T_\circ$- space for $c,d \in X$ such that $c\neq d$ the only $I\alpha$- open set contains c is X but it is also contain d.

Remark 2.8 The $I\alpha$- $T_\circ$- property is not necessary to be topological property, as the following example shows.

Example 2.9 Consider the ideal topological spaces (X,τ,I) and (X,τ,J) such that $X=\{a,b,c,d\}, \tau=\{X,\emptyset,\{a\},\{a,b\}\}, I=\mathcal{P}(X)$ and $J=\{\emptyset\}$.
Then $I\alpha O(X)=\tau$ and $J\alpha O(X)=\{X,\emptyset,\{a\},\{a,b\},\{a,c\},\{a,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\}\}$.

Define a function $f:(X,\tau,I)\longrightarrow(X,\tau,J)$ such that $f(x)=x$, $\forall x\in X$. Then (X,τ,J) is $I\alpha$- $T_\circ$- space but (X,τ,I) is not because, if we take $c,d \in X$ such that $c\neq d$ the only $I\alpha$- open set contains c is X but it is also contain d.

Corollary 2.10 The $I\alpha$- $T_\circ$- property is $I\alpha$- topological property.

Remark 2.11 Every $T_\circ$- space is $I\alpha$- $T_\circ$- space but the converse is not necessary to be true, as the following example shows.

Example 2.12 Take the ideal topological space (X,τ,I) such that $X=\{a,b,c,d\}$, $\tau=\{X,\emptyset,\{a\},\{a,b\}\}$ and $I=\{\emptyset,\{b\}\}$. Then $I\alpha O(X)=\{X,\emptyset,\{a\},\{a,b\},\{a,c\},\{a,b,c\},$

$\{a,b,d\},\{a,c,d\}\}$.

From τ we get that the space X is not $T_\circ$- space but from $I\alpha O(X)$ we get that the space is $I\alpha$- $T_\circ$- space.

Definition 2.13 Let (X,τ,I) be an ideal topological space. Then it is called $I\alpha$- T_1- space if for any two different elements in X there exist two $I\alpha$- open sets in X such that each $I\alpha$- open set of them containing only one element of those elements.

Proposition 2.14 (X,τ,I) is an $I\alpha$- T_1- space if and only if for any element x in X we have that $\{x\}$ are $I\alpha$-closed sets in X.

Corollary 2.15 An ideal topological space (X,τ,I) is an $I\alpha$- T_1- space if and only if the following statements are equivalent:

i. $I\alpha\text{-}cl\{x\} = \{x\}\ ,\forall\, x \in X$.
ii. $\{x\} = \cap\{F: F \in I\alpha C(X) \wedge x \in F\}\ ,\forall\, x \in X$.
iii. $I\alpha\text{-}b\{x\} \subseteq \{x\}\ ,\forall\, x \in X$.
iv. $I\alpha\text{-}d\{x\} \subseteq \{x\}\ ,\ \forall\, x \in X$.
v. $I\alpha\text{-}d\{x\} = \emptyset\ ,\ \forall\, x \in X$.

Corollary 2.16 $I\alpha$- T_1- property is $I\alpha$-hereditary property.

Remark 2.17 $I\alpha$- T_1- property is not topological property, as the following example shows.

Corollary 2.18 $I\alpha$- T_1- property is $I\alpha$- topological property.

Remark 2.19 Every T_1- space is $I\alpha$- T_1- space but the converse is not necessary to be true. As the following example shows:

Example 2.20 Consider the ideal topological space $(\mathcal{N},\tau,I)$ where $\tau = \{u_n \subseteq \mathcal{N}: u_n = \{n, n+1, \dots\}, n \in \mathcal{N}\} \cup \{\emptyset\}$ and $I = \{\emptyset\}$, then the family of all $I\alpha$- open subsets of $\mathcal{N}$ is $\{A \cup u_n: A \subseteq \mathcal{N} \text{ and } u_n \in \tau - \{\emptyset\}\} \cup \{\emptyset\}$. It is clear that $(\mathcal{N},\tau,I)$ is $I\alpha$- T_1- space which is not T_1- space since, if we fix $x = 1$ then for any element $y \in \mathcal{N}$ there is no open set v containing 1 but not contains y.

Remark 2.21 Every $I\alpha$- T_1- space is $I\alpha$- $T_\circ$- space but the converse is not necessary to be true, as the following example shows.

Example 2.22 The (X,τ,I) in Example 2.12 is $I\alpha$- $T_\circ$- space but it is not $I\alpha$- T_1- space for $a, b \in X$ such that $a \neq b$ there is no two $I\alpha$- open sets in X such that each $I\alpha$- open set of them containing only one element of those elements.

Remark 2.23 $I\alpha$- T_1- property is not hereditary property.

Definition 2.24 Let (X,τ,I) be an ideal topological space. It is called $I\alpha$- T_2- space if for any two different elements in X there exist two disjoint $I\alpha$- open sets in X such that each $I\alpha$- open set of them containing only one element of those elements.

Remark 2.25 $I\alpha$- T_2- property is not hereditary property.

Corollary 2.26 $I\alpha$- T_2- property is $I\alpha$- hereditary property.

Remark 2.27 $I\alpha$- T_2- property is not topological property.

Corollary 2.28 $I\alpha$- T_2- property is $I\alpha$- topological property.

Remark 2.29 Every T_2- space is $I\alpha$- T_2- space but the converse is not necessary to be true.

Remark 2.30 Every $I\alpha$- T_2- space is $I\alpha$- T_1- space is $I\alpha$- $T_\circ$- space but the converse is not necessary to be true.

Example 2.31 The (X,τ,I) in Example 2.12 is $I\alpha$- $T_\circ$- space but it is not $I\alpha$- T_2- space for $a, b \in X$ such that $a \neq b$ there is no two disjoint $I\alpha$- open sets in X such that each $I\alpha$- open set of them containing only one element of those elements. the ideal topological space in Example 2.21 is $I\alpha$- T_1- space but not $I\alpha$- T_2- space for any two $I\alpha$- open sets in $\mathcal{N}$, the intersection of them is not empty set.

Definition 2.32 Let (X,τ,I) be an ideal topological space. It is called $I\alpha$- regular space if for every element in X and $I\alpha$- closed set in X does not contain the previous element then there exist two disjoint $I\alpha$- open sets in X such that one of them containing the element and the other set containing the $I\alpha$- closed set. It is denoted by $I\alpha$-R-space.

Definition 2.33 Let (X,τ,I) be an ideal topological space. It is called almost $I\alpha$- regular space if for every element in X and every closed set in X does not contain the previous element then there exist two disjoint $I\alpha$- open sets in X such that one of them containing the element and the other set containing the closed set. It is denoted by almost- $I\alpha$-R-space.

Proposition 2.34 (X,τ,I) is an $I\alpha$- R- space if and only if for any element x in X and every $I\alpha$- open set U in X such that $x \in U$ then there exists another $I\alpha$- open set V in X satisfying that $x \in V \subseteq I\alpha\text{-}cl(V) \subseteq U$.

Proposition 2.35 (X,τ,I) is an almost $I\alpha$- R- space if and only if for any element x in X and every open set U in X such that $x \in U$ then there exists another $I\alpha$- open set V in X satisfying that $x \in V \subseteq I\alpha\text{-}cl(V) \subseteq U$.

Remark 2.36 Both $I\alpha$- R- property and almost $I\alpha$- R- property are not hereditary properties.

Corollary 2.37 $I\alpha$- R- property is $I\alpha$- hereditary property.

Remark 2.38 $I\alpha$- R- property and almost $I\alpha$- R- property are not topological property.

Corollary 2.39 $I\alpha$- R- property is $I\alpha$- topological property.

Remark 2.40 Every R- space is almost $I\alpha$- R- space $I\alpha$- R- space but the converse is not necessary to be true.

Definition 2.41 Let (X,τ,I) be an ideal topological space then it is called $I\alpha$- T_3-space if X is $I\alpha$- T_1-space and $I\alpha$- R-space.

Definition 2.42 Let (X,τ,I) be an ideal topological space then it is called almost $I\alpha$- T_3-space if X is $I\alpha$- T_1-space and almost $I\alpha$- R-space.

Proposition 2.43 If the ideal topological space (X,τ,I) is $I\alpha$- T_3-space then (X,τ,I) is $I\alpha$- T_2-space.

Corollary 2.44 Both almost $I\alpha$- T_3- property and $I\alpha$- T_3- property are not hereditary property.

Proof $I\alpha$- R- property, almost $I\alpha$- R- property and $I\alpha$- T_1- property are not hereditary properties.

Corollary 2.54 $I\alpha$- T_3- property is $I\alpha$- hereditary property.

Proof Both $I\alpha$- R- property and $I\alpha$- T_1- property are $I\alpha$- hereditary properties.

Corollary 2.46 Both almost $I\alpha$- T_3- property and $I\alpha$- T_3- property are not topological properties.

Proof $I\alpha$- R- property, almost $I\alpha$- R- property and $I\alpha$- T_1- property are not topological properties.

Corollary 2.47 $I\alpha$- T_3- property is $I\alpha$- topological property.

Proof Both $I\alpha$- R- property and $I\alpha$- T_1- property are $I\alpha$- topological properties.

Remark 2.48 Every T_3- space is almost $I\alpha$- T_3- space is $I\alpha$- T_3- space but the converse is not necessary to be true.

Definition 2.49 Let (X,τ,I) be an ideal topological space then it is called $I\alpha$- normal space if for every two disjoint $I\alpha$- closed sets in X there exist two disjoint $I\alpha$- open sets in X such that each $I\alpha$- open set contain only one of the two disjoint $I\alpha$- closed sets in X. It is denoted by $I\alpha$-N-space.

Definition 2.50 Let (X,τ,I) be an ideal topological space then it is called almost $I\alpha$- normal space if for every two disjoint closed sets in X there exist two disjoint $I\alpha$- open sets in X such that each $I\alpha$- open set contain only one of the two disjoint closed sets in X. It is denoted by almost $I\alpha$-N-space.

Proposition 2.51 (X,τ,I) is an $I\alpha$- N- space if and only if for any $I\alpha$- closed set F in X and every $I\alpha$- open set U in X containing F then there exists another $I\alpha$- open set V in X satisfying that $F \subseteq V \subseteq I\alpha\text{-}cl(V) \subseteq U$.

Proposition 2.52 (X,τ,I) is almost $I\alpha$- N- space if and only if for any closed set F in X and every open set U in X containing F then there exists another $I\alpha$- open set V in X satisfying that $F \subseteq V \subseteq I\alpha\text{-}cl(V) \subseteq U$.

Remark 2.53 Both $I\alpha$- N- property and almost $I\alpha$- N- property are not hereditary properties.

Example 2.54 Consider the ideal topological space (X,τ,I) such that $X = \{1,2,3,4\}, \tau = \{X,\emptyset,\{2\},\{1,2\},\{2,3\},\{1,2,3\}\}$ and $I = \mathcal{P}(X)$. Then $I\alpha O(X) = \tau$. It is clear that (X,τ,I) is $I\alpha$- N- space and almost $I\alpha$- N-space but if we take $A = \{1,2,3\} \subseteq X$ then (A,τ_A,I_A) is neither $I\alpha$-N- space nor almost $I\alpha$-N- space for $\tau_A = I\alpha O(A) = \{A,\emptyset,\{2\},\{1,2\},\{2,3\}\}$, then $\{1\},\{3\} \in F_A = I\alpha C(A)$ such that $\{1\} \cap \{3\} = \emptyset$ and $\{1\} \subseteq \{1,2\} \in I\alpha O(A), \{3\} \subseteq \{2,3\} \in I\alpha O(A)$ but $\{1,2\} \cap \{2,3\} \neq \emptyset$.

Corollary 2.55 $I\alpha$- N- property is $I\alpha$- hereditary property.

Remark 2.56 Both $I\alpha$- N- property and almost $I\alpha$- N- property are not topological property.

Corollary 2.57 $I\alpha$- N- property is $I\alpha$- topological property.

Remark 2.58 Every N- space is almost $I\alpha$- N- space $I\alpha$- N- space but the converse is not necessary to be true.

Definition 2.59 Let (X,τ,I) be an ideal topological space. It is called $I\alpha$- T_4-space if X is $I\alpha$- T_1-space and $I\alpha$- N-space.

Definition 2.60 Let (X,τ,I) be an ideal topological space. It is called almost $I\alpha$- T_4-space if X is $I\alpha$- T_1-space and almost $I\alpha$- N-space.

Proposition 2.61 Every $I\alpha$- T_4-space is $I\alpha$- R-space.

Proposition 2.62 Every $I\alpha$- T_4-space is $I\alpha$- T_3-space.

Remark 2.63 Both almost $I\alpha$- T_4- property and $I\alpha$- T_4- property are not hereditary properties.

Proof $I\alpha$- N- property, almost $I\alpha$- N- property and $I\alpha$- T_1- property are not hereditary properties.

Corollary 2.64 $I\alpha$- T_4- property is $I\alpha$- hereditary property.

Remark 2.65 Both almost $I\alpha$- T_4- property and $I\alpha$- T_4- property are not topological properties.

Proof $I\alpha$- N- property, almost $I\alpha$- N- property and $I\alpha$- T_1- property are not topological properties.

Corollary 2.66 $I\alpha$- T_4- property is $I\alpha$- topological property.

Remark 2.67 Every T_4- space is almost $I\alpha$- T_4- space is $I\alpha$- T_4- space but the converse is not necessary to be true.

3. $I\alpha$- COMPACT SPACES

In this section we will study $I\alpha$- compact spaces using the concept $I\alpha$- open sets.

Definition 3.1 A subset A of an ideal topological space (X, τ, I) is said to be $I\alpha$- compact if for every cover $\{u_\alpha : \alpha \in \Lambda\}$ by $I\alpha$- open sets in X for A, there is a finite subfamily $\Lambda_\circ$ of Λ such that $A - \cup\{u_\alpha : \alpha \in \Lambda_\circ\} \in I$. A space (X, τ, I) is $I\alpha$- compact if X is $I\alpha$- compact as a set.

Remark 3.2 $I\alpha$- compact property is not hereditary property.

Example 3.3 Consider the ideal topological space (X, τ, I) such that $X = \mathrm{N} \cup \{0, -1\}, \tau = \wp(\mathrm{N}) \cup \{H \subseteq \mathrm{N}: H^c \text{ is finite}, 0 \in \mathrm{H} \text{ or} -1 \in \mathrm{H}\}$and $I = \{\emptyset\}$. Then $I\alpha O(X) = \tau$. The ideal topological space (X, τ, I) is an $I\alpha$- compact space and by taking $A = \mathrm{N} \subseteq X$ which is not $I\alpha$- compact space for: $\{\{n\}: n \in \mathrm{N}\}$ is $I\alpha$ open cover for N but $\mathrm{N} - \cup\{1,2, \dots, \mathrm{m}\} = \text{infinite set} \notin I$.

Remark 3.4 $I\alpha$- compact property is neither topological property nor $I\alpha$- topological property.

Example 3.5 Consider the ideal topological spaces (X, τ, I) and (Y, σ, J) such that $X = Y = \mathrm{N}, \tau = \sigma = D, I = D$ and $J = \emptyset$. Then $I\alpha O(X) = I\alpha O(Y) = D$.

Define a function $f: (X, \tau, I) \longrightarrow (Y, \sigma, J)$ such that $f(a) = a \ \ \forall a \in X$ thus f is continuous, 1-1, onto function and f^{-1} is also continuous. (X, τ, I) is $I\alpha$- compact space but (Y, σ, J) is not. We can see that f is also $I\alpha$- irresolute function and f^{-1} is $j\alpha$- irresolute function. Hence $I\alpha$- compact property is neither topological nor $I\alpha$- topological property.

Corollary 3.6 If $A, B \subseteq X$ such that A, B are $I\alpha$- compact sets over X then $A \cup B$ is also $I\alpha$- compact set.

Remark 3.7 If $A, B \subseteq X$ such that A, B are $I\alpha$- compact sets over X then it is not necessary that $A \cap B$ to be $I\alpha$- compact set. As the following example shows.

Example 3.8 Consider the ideal topological space (X, τ, I) such that $X = \mathrm{N} \cup \{0, -1\}, \tau = \wp(\mathrm{N}) \cup \{H \subseteq \mathrm{N}: H^c \text{ is finite}, 0 \in \mathrm{H} \text{ or} -1 \in \mathrm{H}\}$ and $I = \{\emptyset\}$. Then $I\alpha O(X) = \tau$. The ideal topological space (X, τ, I) is an $I\alpha$- compact space and by taking $A = \mathrm{N} \cup \{0\}, B = \mathrm{N} \cup \{-1\}$ so A, B are $I\alpha$- compact sets over X but $A \cap B = \mathrm{N}$ which is not $I\alpha$- compact set.

Remark 3.9 There is no relation between being the ideal topological space (X, τ, I) compact space or $I\alpha$- compact space.

Example 3.10

i. Consider the ideal topological space (X, τ, I) such that $X = \mathrm{N}, \tau = D$ and $I = \wp(\mathrm{N})$. Then $I\alpha O(X) = D$. We know that N is not compact space but the ideal topological space (N, D, I) is $I\alpha$- compact space.

ii. Consider the ideal topological space (X, τ, I) such that $X = \mathrm{N}, \tau = \{\mathrm{N}, \emptyset, \{1\}\}$ and $I = \{\emptyset\}$. Then $I\alpha O(X) = \{A \subseteq \mathrm{N}: 1 \in \mathrm{A}\} \cup \{\emptyset\}$. We know that N is compact space but the ideal topological space (N, τ, I) is not $I\alpha$- compact space. Now, take $\{\{1, n\}: n \in \mathrm{N}\}$ is an $I\alpha$- open cover for N, then there exists $m \in \mathrm{N} \text{ such that } \cup\{1, \mathrm{m}\}$ is finite subfamily of $\{1, n\}$ but $\mathrm{N} - \cup\{1, \mathrm{m}\} \notin I$, and it is infinite. Hence (N, τ, I) is not $I\alpha$- compact space but (N, τ) is compact.

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